Three Essays on Empirical Asset Pricing and Systematic Ambiguity

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Introduction

Empirical asset pricing faces a number of challenges when predictions of theoretical models fail to pass empirical tests, thus creating puzzles. Attempts to explain stylized facts drive research efforts in asset pricing, portfolio choice theory, and behavioral finance. Classical asset pricing theory relates prices of financial assets to the stochastic discount factor. Therefore, understanding the forces that determine the structure of stochastic discount factors and the investor's rewards for risk taking is instrumental in asset pricing and thus can help resolve empirical controversies.

The aim of my dissertation is to discover new systematic risk factors in various areas of research: specifically, optimal contract theory, hedge fund research, and convertible bond pricing. The first two chapters are devoted to ambiguity aversion, a concept that postulates aversion not only to the uncertain realization of events (known unknowns) but also to the unknown probability measures associated with uncertain external events (unknown unknowns). Following the literature on robust optimization, we construct the ambiguity-averse preferences and the Max-Min utility optimization problem that yields the optimal asset allocation solution for an ambiguity-averse agent. In the first chapter, we demonstrate the implications of the manager's ambiguity aversion for optimal compensation contracts. In the second chapter, we apply an empirically constructed ambiguity factor to hedge fund portfolio allocation. The third chapter examines the systematic effect of hedge fund demand on convertible bond pricing and proposes a price pressure risk factor for the convertible bond market.

The first chapter of the dissertation examines the optimal executive compensation policy when a manager is ambiguity-averse. Standard principal-agent models of optimal contract design poorly explain existing practices of executive compensation, in particular the prevalence of stock-based compensation. This paper addresses this inadequacy by modeling the optimal contract for an ambiguity-averse manager in a continuous-time moral hazard model. The model predicts that the ambiguity-averse manager undertakes less risky projects and exerts lower effort than the risk-averse manager. The optimal contract for the ambiguity-averse manager therefore contains a larger fraction of the firm's equity to mitigate ambiguity aversion and to provide stronger incentives. The paper compares compensation plans consisting of stocks or stock options. We evaluate equity-based compensation from the manager's point of view (subjective evaluation) and from the shareholder's point of view (objective evaluation). The findings in the paper reveal that the manager's ambiguity aversion decreases the subjective value of equity-based compensation and widens the gap between the subjective value and the objective (or market) value. Numerical simulations support the conclusion that stock option holdings in the optimal contract have advantages over stocks in terms of incentive costs to shareholders and additional risk-taking incentives.

The second chapter presents a hedge fund portfolio choice model for an investor facing ambiguity. The investor faces both idiosyncratic hedge fund ambiguity and aggregate market (stock market or macroeconomic) ambiguity. The optimal hedge fund asset allocation model reveals that firstly an investor tends to reduce her allocation to risky assets under ambiguity, hedge funds or stocks alike, and secondly only systematic ambiguity is priced in equilibrium. Moreover, the more directional (strongly correlated with the market) the hedge fund strategy is, the lower the optimal allocation to hedge funds by an investor who is sensitive to ambiguity. The theoretical model derives the equilibrium two-factor capital asset pricing model under ambiguity (ACAPM) that demonstrates how the ambiguity factor is priced in financial markets. It contributes to the alpha versus alternative beta debate by postulating the hypothesis that expected hedge fund returns embed a risk premium for systematic ambiguity exposure. In the empirical section, we measure ambiguity as the cross-sectional dispersion in survey-based macroeconomic forecasts for growth in the Industrial Production Index and in stock market forecasts for S&P 500 Index returns, and we construct the systematic ambiguity factors from the universe of S&P 500 stocks. We estimate ambiguity betas for long/short equity hedge fund strategies and document significant ambiguity exposures, especially for directional long/short equity hedge funds. We compare the out-of-sample performance of portfolios constructed according to the hedge fund alphas' ranking with and without systematic ambiguity exposures and find that the former outperform the latter.

The third chapter addresses an anomaly in convertible bond underpricing by investigating the role of convertible arbitrage hedge funds as liquidity providers to the convertible bond market. Prices of convertible bonds are sensitive to unexpected demand shocks generated by convertible arbitrage hedge funds. This paper investigates whether price pressure created by innovations in hedge fund demand can account for the systematic mispricing of convertible bonds. We test the hypothesis that the mispricing of convertible bonds disappears after taking into account price-pressure risk. We empirically construct the risk factor associated with hedge fund price pressure and document the non-negligible risk premium embedded in convertible bond returns. Moreover, we demonstrate that the contemporaneous returns of convertible bond mutual funds have significant negative exposure to the price-pressure factor. The price-pressure factor provides an incremental improvement over conventional equity and bond factors in explaining returns of convertible bond mutual funds. Price-pressure risk is amplified during financial crises, creating the risk of a convertible bond sell-off.

Contents

Chapter 1

Equity-Based Compensation in the Presence of Ambiguity Aversion

Abstract

Standard principal-agent models of optimal contract design poorly explain existing practices of executive compensation, in particular the prevalence of stock-based compensation. This paper addresses this inadequacy by analyzing the optimal contract for an ambiguity-averse manager in a continuous-time moral hazard model. We study the impact of ambiguity aversion on the optimal structure of managerial compensation plans. The model predicts that an ambiguity-averse manager undertakes less risky projects and exerts lower effort than a risk-averse manager. The optimal contract for the ambiguity-averse manager therefore contains a larger fraction of the firm's equity to mitigate ambiguity aversion and to provide stronger incentives. The paper compares compensation plans consisting of stocks or stock options. The findings of the paper reveal that stock option holdings in the optimal contract have advantages over stocks in terms of incentive costs to shareholders and additional risk-taking incentives.

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1.1 Introduction

Over the last few years, a major debate regarding the optimality of stock options in executive compensation plans has taken place. The options were granted not only to align the interests of managers and shareholders but also, in contrast to linear stock pay, to provide risk-taking incentives. Executive stock option holdings started growing in the 1980s in the United States and became widespread worldwide in the 1990s. As Murphy (1999) reports, 39% of CEO compensation in S&P 500 firms was granted in the form of stock options in 1996, growing to 47% in 1999. These recent trends in compensation patterns have not been successfully predicted by standard principal-agent models of optimal contract design.

Two explanations exist for this discrepancy: either the observed contracts are not optimal or the models are wrong. To support the former hypothesis, Bebchuk and Fried (2004) suggest that the existing compensation structure mainly reflects rent-seeking and thus stays far from being an optimal contract. To support the latter explanation, Dittmann and Maug (2007) argue that the standard principal-agent model used in the compensation literature predicts almost no stock option holding, which contradicts the empirical evidence. An excessive CEO payout, especially in the form of stock options, can be caused by a lower value of executive stock options relative to the market option value due to an undiversified home-biased managerial portfolio and a suboptimal exercise policy. Moreover, large grants of deeply out-of-the-money options make the compensation less valuable at the date of grant.

This paper proposes an alternative explanation for the prevalence of equity-based executive compensation. We introduce an ambiguity-averse manager who takes into account possible model misspecification in her decision-making process. The paper addresses the importance of the manager's ambiguity aversion in her evaluation of her stock option compensation. The ambiguity-averse manager has lower risk-taking incentives as she is averse to both risk and ambiguity. Hence the optimal compensation contract for ambiguity-averse executives should contain stronger risk-taking incentives, mitigating ambiguity aversion. As a result, the optimal contract structure should favor equity-based compensation, and in particular, larger stock option holdings.

A manager is generally exposed to several sources of ambiguity. First, a CEO might be averse to the idiosyncratic source of ambiguity because she has imperfect control of the firm's performance as measured by cash flow dynamics. Second, there exists ambiguity about the market risk and systematic factors that explain the stock price in a multifactor asset-pricing model. Moreover, ambiguity is also caused by the model risk reflected in the uncertainty in the set of systematic factors of the asset-pricing model. We show in this paper that the manager's ambiguity aversion contributes to the abnormal returns on the firm's stock.

This paper offers a theoretical model of the optimal contract for an ambiguity-averse manager in the principal-agent framework. The manager has control over the underlying stock price process via her choice of idiosyncratic volatility and effort. From this perspective, the specification of control variables of managers is similar to the case in Cadenillas et al. (2004). Effort is costly as in a standard moral hazard model, but there is no disutility in choosing volatility. The choice of volatility reflects the managerial choice of risky projects. The manager's problem is a version of Merton's consumption and investment choice problem solved by robust optimization techniques. However, the investment choice is constrained by the compensation contract. The constraint determining the firm's stock loading in the managerial portfolio is endogenous and chosen by a principal. A shareholder acts as a principal and maximizes the firm's expected value by setting the firm's equity value in the compensation package. The compensation plan for the manager consists of either the firm's stock or stock options.

The paper solves the model for the optimal effort, risk choice, consumption, and equity compensation and studies the effect of ambiguity aversion on the optimal solution. Furthermore, the paper calibrates the model parameters to provide a comparative statics analysis of the implicit optimal solutions. The ambiguity-averse agent has a higher overall risk aversion, implying the choice of less risky projects. The key observation of the optimal solution is that ambiguity aversion reduces the optimal risk choice and the optimal effort of the manager. The ambiguity-averse manager therefore has lower risk-taking incentives and lower incentives generally than a risk-averse manager. As a result, shareholders should increase these incentives via higher volumes of stock-based compensation in order to mitigate the effect of ambiguity aversion. This effect is consistent with the results of Uppal and Wang (2003) who explain the home bias in an investor's portfolio by ambiguity aversion.

Ambiguity aversion also reduces the subjective value of equity-based compensation contracts. The subjective evaluation — the evaluation from the manager's point of view — is based upon work by Ingersoll (2006), who suggests that a manager evaluates the firm's stocks or stock options at lower values than the market does. Our paper finds that ambiguity further decreases the subjective value of equity-based compensation for a manager and widens the gap between the subjective value and the market value. This gap is a deadweight loss for a company. Our paper compares evaluations of stocks and stock options and concludes that both forms of compensation can deliver required incentives for the ambiguity-averse manager. However, executive stock options are superior to stocks in terms of incentive costs to shareholders and ability to provide risk-taking incentives.

The remainder of the paper is organized as follows. Section 1.2 briefly describes the relevant literature on executive compensation and ambiguity aversion in finance, and outlines how this paper stands in the literature. Section 1.3 describes the setup of the principal-agent model of executive compensation contract and introduces the ambiguity-averse preferences of the manager. Section 1.4 presents the optimal solution of the optimization problem, the subjective evaluation of equity-based compensation, and numerical simulations. Concluding remarks and future research avenues are provided in Section 1.5. The appendices in Section 1.6 contain all proofs, derivations, and tables.

1.2 Literature Review

Holmstrom and Milgrom (1987), in their seminal paper on continuous-time principal-agent models, demonstrated the linearity of the optimal contract when both the agent and the principal have exponential utilities. Sung (2005) has extended this pure moral hazard model as well as his own earlier papers (Schattler and Sung (1993) and Sung (1995)) by incorporating adverse selection into the model. This leads to constant values of optimal controls and linearity of optimal contracts. His setting is characterized by a risk-neutral principal and a risk-averse agent with exponential utility who can control the volatility as well as the drift of the stochastic process for the company's stock. As a solution method he used dynamic programming and the martingale approach of stochastic control theory. Carpenter (2000) also considered a dynamic model with the possibility for the agent to choose volatility. However, the agent cannot affect the drift of the stock price process in her framework. Ou-Yang (2003) solves the Hamilton-Jacobi-Bellman (HJB) equation in the model where an agent can affect both the drift and the diffusion rate simultaneously. This feature allows application of this principal-agent setting to delegated portfolio management problems. Cadenillas et al. (2004) use a stochastic optimization technique and the martingale approach in order to derive the optimal leverage and compensation level chosen by the company and the optimal value for effort and volatility, which are controlled by a manager. This setting can be used to assess the incentive effect of executive stock options. Our paper considers a similar linear specification for effort and volatility of the drift dynamics of the stochastic process describing the company's stock price process; however, we employ the HJB method to obtain the optimal solution.

Both stocks and options have an incentive purpose in the compensation contracts but their effect is quite different. Murphy (1999) and Core et al. (2003) argue that convex payouts such as stock options mitigate the effect of executive risk aversion by giving managers incentives to take riskier projects. Moreover, firms with greater growth opportunities provide higher risk-taking incentives and should therefore offer compensation packages with options. There are also additional features of executive stock options that have been noted in the literature: the value of stock options decreases with the level of dividends, thus producing incentives to reduce dividend payments after granting options to executives; also, as Murphy (1999) reported, executive stock options are likely to favor share repurchases.

Carpenter (2000) investigates the relationship between risk preferences and option compensation by modeling the optimal dynamic investment policy for a manager paid with options on the firm's stock. She found that the optimal compensation policy should contain either deep in- or out-of-the-money options, but the impact of options on risk-taking behavior is not linear. The risk aversion of the manager prevents him from preferring high volatility of the stock price and forces him to choose volatility depending on the asset value. When the asset value is small or options are deep out of the money, it is optimal to increase asset volatility. However, if the asset value becomes large or options are near the money, the manager lowers the risk. Moreover, Carpenter (2000) demonstrates situations when vesting restrictions on executive options motivate the manager to set the volatility of stock prices lower than in the case of tradable options without vesting restrictions. Ross (2004) also argues that giving options to managers will not necessarily stimulate risk-taking behavior, and the incentive effect will depend on risk aversion. Lambert and Larcker (2004) show that the optimal contract should contain out-of-the-money options, and thus the choice of the optimal strike price is important for the incentive effect. Ju et al. (2014) predict that call options may induce suboptimal risk choice for a risk-averse manager. Therefore, the authors propose including lookback call options to ensure ex-post risk-taking incentives and put option-type features to ensure ex-ante risk-taking incentives in managerial contracts. Lookback call options mitigate the practice of option repricing. Put options implicitly represent existing severance packages in the compensation contracts.

Another important issue is the manager's evaluation of stock-based compensation, which tends to be lower than the market value or the objective value for shareholders because of the absence of diversification due to vesting restrictions, a suboptimal exercise policy of executive stock options, and the manager's risk aversion. Ingersoll (2006) investigates the subjective evaluation of an executive stock option in details. Ingersoll (2006) prices the executive options using a risk-neutral probability that corresponds to the constrained optimization problem of a manager. The more risk averse the agent or the stronger the restrictions in place, the smaller the subjective value. The present paper also shows benefits from using restricted stock that are consistent with recent empirical findings. The cost of executive stock options to the shareholder refers to the objective value, which is higher than the subjective value but lower than the market Black-Scholes price. Carpenter (1998) calculates the value of executive stock options as an American option with a vesting restriction and a random exogenous exercise. Lambert et al. (1991) compute the certainty equivalent for the pricing of executive options from a manager's perspective.

Ellsberg's paradox (1961) illustrates the difference in behavior of economic agents under risk and ambiguity and thus emphasizes the importance for economic decision making of accounting for ambiguity. Ambiguity-averse preferences are based on the robust optimization problem with a penalty entropy term. An ambiguity-averse agent minimizes the difference measured by relative entropy between her expected utilities under a reference model and an alternative model subject to a penalty for deviation. In a second step, the agent maximizes her utility function with respect to the control variables. There are two settings of such a Max-Min optimization problem. The first one uses local relative entropy and constrained optimization. This setting corresponds to the axiomatic recursive multiple prior utility setting based on Gilboa and Schmeidler (1989) preferences over a set of multiple prior distributions, further developed by Epstein and Schneider (2003) in discrete time and by Anderson et al. (2003) in continuous time. The second setting is characterized by adding the global relative entropy as a penalty term for deviations into the Max-Min optimization. Maccheroni et al. (2006) build an axiomatic foundation for ambiguity-averse preferences consistent with economic theory for deviations expressed by global relative entropy. A thorough discussion of two different ambiguity-averse settings appears in Trojani and Vanini (2004).

Robust decision making has been widely used in the asset-pricing literature to tackle a variety of puzzles. An important result of observational equivalence in asset pricing was obtained by Maenhout (2004), Uppal and Wang (2003), Anderson et al. (2003), and originally by Duffie and Epstein (1992) for solving a Merton's-type robust portfolio optimization problem. This finding states that taking into account ambiguity aversion increases initial risk aversion on the part of the decision maker. This increase would be one possible explanation of the equity premium puzzle. Dow and Werlang (1992) used ambiguity-averse preferences to generate the limited equity market participation effect in optimal portfolio choice. Another interesting implication of robustness in asset pricing is its ability to tackle the home-bias puzzle in a manager's portfolio as demonstrated by Uppal and Wang (2003). Trojani and Vanini (2004) analyze ambiguity-averse preferences in intertemporal heterogeneous agent economies.

1.3 The Model

This section describes the model of the optimal compensation contract, where the ambiguity-averse manager solves her consumption and investment allocation problem and the shareholder decides upon the manager's compensation plan.

1.3.1 Assets in the Economy

The manager allocates her wealth between risky assets and risk-free bonds. Risky assets are represented by the market index (market portfolio) M_t and the firm's stock S_t. The company's stock is also a part of the market index. Without any constraints, the manager would invest a fraction of her wealth θ_M only in the market portfolio to eliminate idiosyncratic risk. However, the compensation contract requires the manager to hold a proportion of her wealth θ in the firm's stock until the termination of her employment. This fraction θ is measured in excess of the stock's share in the market allocation θ_M. After the termination of the contract, she is free to hold the market portfolio, and she solves the standard portfolio and consumption choice problem as in Merton (1969). We do not consider optimal portfolio choice after the termination of the manager's employment.

The assets in the economy are described as follows:

- risk-free bond
$$dB_t = rB_t dt; \tag{1.1}$$

- market index, which also includes the company's stock:
$$\frac{dM_t}{M_t} = \mu_M dt + \sigma_M dZ_t^M; \tag{1.2}$$

- the firm's stock
$$\frac{dS_t}{S_t} = \mu dt + \sigma\sqrt{1-\rho^2} dZ_t^S + \sigma\rho dZ_t^M, \tag{1.3}$$

where ρ is the correlation coefficient between firm and market index, $\mathbb{E}(dZ_t^S dZ_t^M) = \rho dt$.

It is possible to choose any asset pricing model to explain the expected return of a company's stock. We consider the capital asset pricing model (CAPM) with a possibility of abnormal returns α in order to emphasize the effect of managerial choice of firm-specific risk on expected returns:
$$\mu = r + \frac{\sigma\rho}{\sigma_M}(\mu_M - r) + \alpha. \tag{1.4}$$

The coefficient α is a possible mispricing that is affected by managerial actions, and has important implications for this paper. First, it contains the efforts of the manager because the manager can affect the stock price via exerting higher effort u. Second, α contains the manager's choice of idiosyncratic volatility σ. The volatility σ indicates the choice of a manager within a menu of risky projects with different levels of risk. The choice of σ

affects both the expected returns and the firm-specific risk. We assume that α is a linear function of effort u and firm-specific volatility σ:

$$\alpha = \xi u + \eta\sigma, \text{ where } \xi > 0, \eta > 0. \tag{1.5}$$

The coefficient ξ represents the manager's type. This interpretation was inspired by Cadenillas et al. (2004). The higher ξ, the lower the effort required to achieve the equivalent impact on stock returns. We call the coefficient ξ the effort multiplier.

The coefficient η can be considered as a proxy for an industry classification. We call the coefficient η the growth prospects. High values of η imply a high-growth sector such as the information technology sector, while low values of η imply characteristics of a low-growth sector such as the utilities sector. In this interpretation, the coefficient η is a discrete variable, which ex-ante corresponds to the particular volatility level.

The manager's choice of effort and volatility is an indicator for shareholders to determine whether the manager requires additional incentives to boost her effort level or risk-taking incentives to motivate her to take on a riskier project. In order to achieve a certain target for α, the manager can either work hard without being involved in more profitable but riskier projects, choose a lower effort level and try to make higher returns in riskier investments, or do both.

1.3.2 Introducing Ambiguity

Assume that the model or more precisely the probability law that characterizes the stochastic dynamics of assets returns is not correctly specified. Let $\mathcal{P}$ be a probability measure for the reference model. All deviations from the reference model are described by alternative probability measures $\mathcal{Q}^h$. The aim of the agent optimization is to find the true model specification from among appropriately specified alternatives $\mathcal{Q}^h$.

Let $\mathcal{Q}^h$ be a set of probability measures absolutely continuous with respect to $\mathcal{P}$ and parameterized by an appropriately adapted process h_t[1]. Assume that the Radon-Nikodym derivative of $\mathcal{Q}^h$ with respect to $\mathcal{P}$ is the following:

$$\frac{d\mathcal{Q}^h}{d\mathcal{P}} = \exp\{-\int_0^t \frac{||h_s||^2}{2}ds - \int_0^t h_s dZ_s\}, \tag{1.6}$$

where $Z_t = (Z_t^S, Z_t^M)$ is a vector of Brownian motions and $h_t = (h_t^S, h_t^M)$ is a vector of parameters related to the corresponding source of ambiguity. The agent faces ambiguity with respect to both the idiosyncratic risk of the firm and the market risk. Under Girsanov's Theorem, there exists an adapted process h_t such that $Z_t^h = Z_t + \int_0^t h_s ds$ is a $\mathcal{Q}^h$-Brownian motion. The distorted dynamics of the market index are the following:

$$\frac{dM_t}{M_t} = (\mu_M + \sigma_M h_t^M)dt + \sigma_M dZ_t^M, \tag{1.7}$$

and those of the firm's stock are

$$\frac{dS_t}{S_t} = (\mu + \sigma(\rho h_t^M + \sqrt{1-\rho^2}h_t^S))dt + \sigma(\rho dZ_t^M + \sqrt{1-\rho^2}dZ_t^S). \tag{1.8}$$

The distortion affects the expected returns of the assets, but the volatilities remain unchanged under the alternative probability measures.

The distortion can be formally considered as a part of the mispricing term α. The distorted mispricing term α^h is given by the following equation:

$$\alpha^h = \xi u + \eta\sigma + \sigma(\rho h_t^M + \sqrt{1-\rho^2}h_t^S). \tag{1.9}$$

The ambiguity distortion in the mispricing term is caused by the presence of model risk, namely use of the wrong asset-pricing model and mistakes in factor identifications as well as the imperfect influence of the manager on performance.

[1]In general, $h = h(S_t, t)$ but we consider only time-dependence for the sake of computational simplicity.

Later we will demonstrate that the ambiguity parameters h^M and h^S are negative, and thus concern about ambiguity reduces α and expected stock returns. Moreover, ambiguity diminishes the growth prospects from η to $\eta + (\rho h_t^M + \sqrt{1-\rho^2} h_t^S)$. Ambiguity forces the manager to avoid risky projects. Hence the ambiguity-averse manager is likely to require stronger risk-taking incentives.

1.3.3 The Manager's Problem

The manager solves her consumption-investment constrained optimization problem and also chooses the optimal levels of effort and volatility. Effort u is costly, and its disutility can be expressed by any convex function. For the sake of computational simplicity, we assume a quadratic cost of effort. The marginal benefit of effort is higher for the agent of a higher type ξ. The optimization problem for the standard risk-averse agent is the following:

$$\max_{u,\sigma} \max_{\theta_M, C_t} \mathbb{E} \int_0^\infty e^{-rt} \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \frac{u^2}{2} \right) dt, \tag{1.10}$$

where θ_M is the fraction of wealth invested in the market index, C_t is the manager's instantaneous consumption, r is the risk-free discount rate, u is the choice of effort, and σ is the choice of firm-specific volatility. For computational simplicity, we assume an infinite investment horizon[2] and a zero bequest condition. The instantaneous utility is assumed to be of the constant relative risk-aversion (CRRA) type, where γ is the coefficient of relative risk aversion.[3]

There are two ways to represent the size of the model's misspecification: by means of

[2]The infinite time horizon simplifies the HJB optimality equation: the partial differential equation is reduced to an ordinary differential equation. However, this assumption can be relaxed. The finite-horizon model for the CRRA utility function can also be solved explicitly as shown in Merton (1969).

[3]In the case $\gamma = 1$, the utility has a logarithmic functional form. In this paper, we present results only for the general case $\gamma > 1$.

local relative entropy

$$Ent_t^{loc}(\mathcal{Q}^h|\mathcal{P}) = \mathbb{E}^h \frac{1}{2}\int_0^t h_s^2 ds \tag{1.11}$$

or global relative entropy

$$Ent^{glob}(\mathcal{Q}^h|\mathcal{P}) = r\int_0^\infty e^{-rt} Ent_t^{loc}(\mathcal{Q}^h|\mathcal{P})dt = r\int_0^\infty e^{-rt}\mathbb{E}^h \frac{1}{2}\int_0^t h_s^2 ds. \tag{1.12}$$

Different settings arise because of differences in multiple prior distributions. Recursive multiple-prior utility theory provides axiomatic foundations for an agent's preferences with local relative entropy. However, Maccheroni et al. (2006) have recently developed the axioms for preferences with global entropy penalties as well as for a more general class of functions that can be used as a penalty term.

The ambiguity-averse preferences are represented by the expected utility with an additional penalty term for deviations from the reference model. We use the global relative entropy to define the size of the distortion. The penalty term in the utility function is the global relative entropy scaled by a nonnegative parameter that reflects the manager's ambiguity aversion. The Max-Min optimization of the objective function is a maximization of the expected utility function under the search for the worst-case probabilistic scenario. The manager is uncertain about the true probability measure and averse to this situation. Therefore, she prefers to face the worst-case scenario. The optimization depicts a search for the worst-case scenario by minimizing over a feasible set of alternative probability measures $\mathcal{Q}^h$ with a finite relative entropy or, equivalently, over the ambiguity parameter h_t.

The Max-Min optimization problem for the manager becomes

$$\max_{u,\sigma}\max_{\theta_M,C_t}\min_{h_t}\mathbb{E}^h\int_0^\infty e^{-rt}\left(\frac{C_t^{1-\gamma}}{1-\gamma} - \frac{u^2}{2} + \psi\frac{h_t^2}{2}\right)dt, \tag{1.13}$$

subject to the distorted dynamics of the stock price (1.8) and market index (1.7).[4] The

[4]The expectation in the optimization problem and the stochastic dynamics are under probability mea-

manager selects the optimal allocation of her wealth to consumption and investment in the market index. The optimization does not contain the managerial allocation of the company's stock because it is determined by shareholders. The manager's portfolio is not a diversified market portfolio because she is constrained to hold a certain fraction of wealth in the firm's stock.

The term $\psi\frac{h_t^2}{2}$ is a penalty function expressed by a relative global entropy measure. The minimization with respect to h_t reflects the fact that the manager aims to limit the magnitude of the distortion of the assets' stochastic dynamics caused by ambiguity. The higher the weighting function ψ, the higher the penalty for deviation from the reference model and the lower the ambiguity aversion of the agent. Therefore, $\frac{1}{\psi}$ is proportional to the ambiguity-aversion coefficient. As in Skiadas (2003), this coefficient also serves as a measure of comparative risk-aversion. In addition, ψ corresponds to a monotone increase of utility in information filtration; for example, more and earlier information increases utility. The range of ψ is $[0, \infty]$. The case $\psi = 0$ corresponds to a myopic solution as the manager does not penalize the choice of alternative probability measures. The case $\psi \to \infty$ indicates infinite penalty, and thus she always chooses the reference model. Maenhout (2004) allows ψ to be a function of the continuation utility, and this choice is determined by a relatively easy possibility of obtaining a closed-form solution. We use the same specification of ambiguity:

$$\frac{WJ_W}{\psi} = \Omega, \tag{1.14}$$

where W is managerial wealth, J is the indirect utility function that satisfies the HJB optimality equation[5], and the constant Ω denotes the ambiguity-aversion coefficient.

sure $\mathcal{Q}^h$.

[5]See Appendix 1.6 for details of the HJB equation.

1.3.4 The Firm's Problem

The firm's optimization problem is represented by the board members' decision on a compensation package for executives, which is determined by the fraction θ of managers' wealth that should be invested in the company's stock. In fact, shareholders determine only the lower bound for the firm's holding constraint, $\theta \geq \tilde{\theta}$. However, this lower bound will be binding for the manager because absent constraints a manager should hold only the market portfolio.

The firm maximizes its value, which is identical to the stock price at any time T:

$$\max_{\theta} \mathbb{E} S_T, \tag{1.15}$$

subject to optimal values of effort and firm-specific volatility chosen by the manager. We assume that there is no leverage in the capital structure of the firm. Moreover, the principal cannot observe the manager's effort. Following Dittmann and Maug (2007) and Sung (2005), the shareholder, who is the principal in the model, is assumed to be risk-neutral. Unlike the manager, who is risk-averse and ambiguity-averse, shareholders are only concerned with the firm's expected value. Under this assumption, the principal's evaluation of compensation contracts is the risk-neutral evaluation. Following the terminology of Ingersoll (2006), the shareholders' valuation is the objective valuation, which is higher than the manager's subjective valuation but may be lower than the market valuation. However, we assume that the shareholders' evaluation of the compensation contract coincides with the market evaluation. The compensation horizon T determines the vesting restrictions imposed on the manager's compensation, whether in the form of the firm's stock or stock options. The control variable θ is the fraction of the manager's wealth linked to the firm's equity. The principal selects θ to maximize the firm's expected value.

1.4 Results

The above-stated optimization problem is a Merton's-type problem with additional control variables. The standard stochastic dynamic programming technique that leads to a solution of the HJB optimality equation is applicable to our model. This section presents the optimal solution of the model. Furthermore, we calibrate the parameters of the optimal solution in the case when the compensation contract is defined in terms of stocks and stock options. We compare the effect of the manager's ambiguity aversion on the subjective (for the manager) and objective (for the shareholders) evaluations of the compensation payout in the form of either the firm's stock or stock options. Based on these valuations, shareholders can select the securities to be distributed to the manager.

1.4.1 Optimal Parameter Choice for the Principal and Agent

Theorem 1.1 derives the optimal values of effort and volatility selected by the manager as well as the managerial holding in a firm's equity determined by shareholders. These values are the solutions to the manager's problem (1.13) and the shareholders' problem (1.15). In addition, Theorem 1.1 provides the optimal value of consumption and the optimal investment in the market portfolio.

Theorem 1.1. *Depending on the equity compensation θ, the optimal choice of risk σ^* for the manager is the following:*

$$\sigma^* = \frac{\eta + \frac{\rho\Omega(\mu_M - r)}{\sigma_M(\Omega+\gamma)}}{\theta\left(\gamma + \Omega(\rho + \sqrt{1-\rho^2})^2\right)\left(1 - \frac{\gamma^2\rho^2}{(\gamma+\Omega)(\gamma+\Omega(\rho+\sqrt{1-\rho^2})^2)}\right)}, \tag{1.16}$$

and the optimal choice of effort u^* for the manager is the following:

$$u^* = \theta\xi a^{-\gamma} W_0^{1-\gamma}, \tag{1.17}$$

where a is a solution of the equation

$$a = r + \frac{\gamma - 1}{2\gamma}\theta^2 \xi^2 a^{-\gamma} W_0^{1-\gamma} + y, \tag{1.18}$$

and y depends only on risk and ambiguity preferences, the market risk premium, the risk-free interest rate, and the correlation coefficient ρ.

The optimal fraction θ^* of managerial wealth held in the firm's stock in excess of the market portfolio satisfies the following equation:

$$\theta^* = a^{\frac{\gamma}{2}} \xi W_0^{\frac{\gamma-1}{2}} \left(\eta + \frac{\rho(\mu_M - r)}{\sigma_M} \right)^{\frac{1}{2}}. \tag{1.19}$$

The optimal consumption of the manager is given by

$$C_t^* = aW_t. \tag{1.20}$$

The optimal fraction of wealth invested in the market portfolio is the following:

$$\theta_M^* = \frac{\mu_M - r - \gamma\theta^* \beta \sigma_M^2}{\sigma_M^2(\gamma + \Omega)}, \tag{1.21}$$

where $\beta = \frac{\sigma\rho}{\sigma_M}$ is the market beta.

The comparative statics analysis of the optimal parameters is complicated due to the presence of implicit equations. Here, we comment on explicit links between the manager's optimal choice and the model parameters. (The impact of ambiguity aversion on the optimal solution is the subject of numerical simulations presented in Section 1.4.3.)

First, the consumption and investment choice of the manager follows a Merton's solution of the portfolio optimization problem. Optimal consumption is a linear function of wealth. The optimal fraction of wealth invested in the market portfolio is a Merton's portfolio weight adjusted for ambiguity aversion Ω and excess holding θ of the firm's stock.

Ambiguity aversion decreases the investment in risky assets and amplifies the overall effect of risk aversion. The adjustment for θ reflects the fact that the manager holds an undiversified portfolio constrained by mandatory holdings in the firm's equity as defined by her equity-based compensation contract.

Second, one observes from the formula for the optimal effort (1.17) that a manager of a higher type (i.e., a higher effort multiplier ξ) exerts higher effort in equilibrium.

Third, the positive linear relationship between the growth prospects η and the firm-specific volatility σ in equation (1.16) is consistent with the empirical fact that high-growth firms are prone to take on more risk. Moreover, a larger managerial holding θ in the firm's stock results in lower risk.

Finally, ambiguity aversion as well as risk aversion should decrease the manager's likelihood of accepting riskier projects. From the shareholder's point of view, the drop in firm-specific volatility caused by ambiguity aversion diminishes his utility because the firm's value is positively related to σ and shareholders are risk-neutral. Therefore, shareholders should use a higher share θ of equity compensation to induce the ambiguity-averse manager to maintain the same level of incentive as a risk-averse manager. Whether this effect is present in equilibrium is illustrated by the numerical simulations in Section 1.4.3.

1.4.2 Effect of Ambiguity on Option and Stock Compensation

This section investigates the impact of ambiguity on the optimal compensation paid either in shares of the firm's stock or in stock options. We evaluate the equity compensation from the shareholders' point of view (objective evaluation) and from the manager's point of view (subjective evaluation).

The objective evaluations of assets at time T are identical to their expected market values, where expectations are taken under the reference probability measure.[6] Although

[6] In the general case, the value of option compensation to shareholders may be less than its market value because of possible suboptimal exercise policy. However, we ignore this argument and assume that

the shareholders' objective evaluations are computed using expected returns μ as defined in equation (1.4), the manager's ambiguity aversion still affects the compensation evaluation due to optimal choice of effort u^* and volatility σ^*. Theorem 1.2 states that the objective value of stock compensation is the present value of holding the stock until time T, and the objective value of stock option compensation is given by the Black-Scholes model value with a strike price K and a time to maturity T. The incentive effect of option-based compensation is measured by the option delta, the sensitivity of the option value with respect to the underlying stock's price. The convexity of the compensation payoff is given by the option gamma, the sensitivity of the option delta with respect to the underlying stock's price. The risk-taking incentive is given by the option vega, the sensitivity of the option value to volatility.

Theorem 1.2. *The objective value of the firm's stock held until time T is the following:*

$$V_S^M = \mathbb{E}S_T = S_0 e^{\mu T}, \tag{1.22}$$

where μ is given by equation (1.4) and evaluated at the optimal values of u^* and σ^*. The incentive effect of stock compensation is expressed by $\Delta_S^M = e^{\mu T}$.

The objective value of the stock option compensation is the following:

$$V_O^M = S_0 e^{\alpha T}\mathcal{N}(d_1) - Ke^{-rT}\mathcal{N}(d_2), \tag{1.23}$$

where T is the time to maturity, α is given by equation (1.5) ($\alpha = \xi u + \eta\sigma$), K is the strike price, $\mathcal{N}$ is the standard normal cumulative distribution function, and $d_{1,2} = \frac{\ln(S_0/K)+(r+\alpha\pm\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$.

The incentive effect is measured by the option delta:

$$\Delta_O^M = e^{\alpha T}\mathcal{N}(d_1). \tag{1.24}$$

the objective value of option compensation to shareholders equals the Black-Scholes market value.

Option convexity is measured by the option gamma:

$$\Gamma^M = \frac{e^{\alpha T}}{S_0\sigma\sqrt{T}}\frac{\partial\mathcal{N}(d_1)}{\partial d_1}. \tag{1.25}$$

The risk-taking incentive is measured by the option vega:

$$\nu^M = S_0\sqrt{T}e^{\alpha T}\frac{\partial\mathcal{N}(d_1)}{\partial d_1} - \eta e^{\alpha T}\mathcal{N}(d_1). \tag{1.26}$$

The subjective evaluation of compensation is defined from the manager's point of view. First, the ambiguity-averse manager computes expected asset values under an alternative probability measure. The manager's ambiguity aversion affects the expected stock return μ (1.4) via ambiguity distortion in the mispricing term (1.9):

$$\mu^h = \xi u + \eta\sigma - \sigma\Omega(\rho\sigma_M\theta_M + \theta\sqrt{1-\rho^2}(\rho + \sqrt{1-\rho^2})) + r + \frac{\sigma\rho}{\sigma_M}(\mu_M - r). \tag{1.27}$$

The manager evaluates compensation contracts using the distorted stock dynamics μ^h (1.27) under the alternative probability measure and taking optimal values of effort u^* and volatility σ^*. Note that ambiguity aversion affects μ not only via the explicit additional term α^h but also implicitly via the equilibrium values of σ^*, u^*, and θ^*.

The second argument in the subjective evaluation of equity-based compensation is based on Ingersoll (2006). The subjective evaluation is lower than the objective evaluation due to an undiversified managerial portfolio that is constrained by the compensation contract to hold the firm's stocks or stock options. According to Ingersoll (2006), subjective evaluations affect both interest rate and risky cash flows; in other words, the dividend yield. The subjective interest rate $\tilde{r}$ is lower than the market interest rate r while the subjective dividend yield is higher than the actual dividend yield. We apply this framework to the setting with an ambiguity-averse manager.

Theorem 1.3 presents formulas for subjective evaluations of stocks and stock options held until time T by the ambiguity-averse manager. Furthermore, we compute option greeks such as delta, vega, and gamma to measure the incentive and the risk-taking incentive effects.

Theorem 1.3. *The subjective value of the firm's stock held until time T by the ambiguity-averse manager is the following:*

$$V_S^S = e^{-(\gamma(1-\theta)\theta\sigma^2(1-\rho^2))T} S_0. \tag{1.28}$$

The incentive effect of stock compensation (the stock's subjective delta) equals

$$\Delta_S = e^{-(\gamma(1-\theta)\theta\sigma^2(1-\rho^2))T}. \tag{1.29}$$

The subjective value of the stock option is the following:

$$V_O^S = S_0 e^{-T\tilde{q}} \mathcal{N}(\widetilde{d}_1) - K e^{-T\tilde{r}} \mathcal{N}(\widetilde{d}_2), \tag{1.30}$$

where T is the time to maturity, K is the strike price, and $\widetilde{d}_{1,2} = \frac{\ln(S_0/K)+(\tilde{r}-\tilde{q}\pm\frac{\sigma^2}{2}T)}{\sigma\sqrt{T}}$, where

$$\tilde{r} = r - \gamma\theta^2\sigma^2(1-\rho^2), \tag{1.31}$$

$$\tilde{q} = -\alpha^h + \gamma(1-\theta)\theta\sigma^2(1-\rho^2), \tag{1.32}$$

$$\alpha^h = \xi u + \eta\sigma - \sigma\Omega(\rho\sigma_M\theta_M + \theta\sqrt{1-\rho^2}(\rho + \sqrt{1-\rho^2})). \tag{1.33}$$

The incentive effect is measured by the subjective option delta:

$$\Delta_O^S = e^{-T\tilde{q}} \mathcal{N}(\tilde{d}_1). \tag{1.34}$$

The subjective option gamma is the following:

$$\Gamma^S = \frac{e^{-T\tilde{q}}}{S_0 \sigma \sqrt{T}} \frac{\partial \mathcal{N}(\widetilde{d}_1)}{(\widetilde{d}_1)}. \tag{1.35}$$

The risk-taking incentives are measured by the subjective option vega as follows:

$$\begin{aligned} \nu^S &= -e^{\alpha^h T} \mathcal{N}(d_1^h) \left(\eta - \Omega \left(\rho \sigma_M \theta_M + \sqrt{1-\rho^2}(\rho + \sqrt{1-\rho^2})\theta \right) - 2\gamma\theta\sigma(1-\rho^2) \right) \\ &\quad + S_0 \sqrt{T} e^{\alpha^h T} \frac{\partial \mathcal{N}(d_1^h)}{\partial d_1^h}. \end{aligned} \tag{1.36}$$

The subjective evaluation coupled with ambiguity aversion lowers the equity compensation values and affects the incentives provided by stocks or stock options. Section 1.4.3 numerically analyzes this impact. Moreover, we measure shareholders' cost of providing incentives as a market compensation value per unit of subjective delta. The stock incentive costs are computed as follows:

$$SIC = \frac{V_S^M}{\Delta_S}, \tag{1.37}$$

and the option incentive costs equal the following:

$$OIC = \frac{V_O^M}{\Delta_O^S}. \tag{1.38}$$

1.4.3 Numerical Simulations

This section calibrates the model parameters to illustrate numerically the optimal solution, in particular, its sensitivity to different values of ambiguity aversion and risk aversion, and provides a comparative statics analysis. The results of this study provide guidance on ways to alter the compensation package to mitigate the ambiguity effect.

The following numerical parameters are fixed. The market is characterized by volatility $\sigma_M = 0.3$ and an expected rate of return $\mu_M = 10\%$. The risk-free rate is assumed to be

$r = 4\%$. The initial value of the firm's stock price is $S_0 = 1$. The firm's stock does not pay dividends. The initial wealth of the manager is normalized to one, $W_0 = 1$. Finally, the growth prospects coefficient is $\eta = 0.5$ and the effort multiplier $\xi = 0.5$. The time horizon is taken to be one year, $T = 1$.

The comparative statics are based on variation of the risk-aversion coefficient $\gamma \in \{2, 3, 4\}$ and the ambiguity-aversion coefficient $\Omega \in \{0.01, 0.5, 1\}$.[7] The case $\Omega = 0.01$ represents the reference model without manager ambiguity concerns. In addition, the correlation coefficient between the stock return and the market return takes either of two values $\rho \in \{0.1, 0.4\}$.

First of all, we are interested in the optimal value of volatility selected by an ambiguity-averse manager given some pre-specified equity compensation θ. We compare two compensation grants with 30% or 60% of the wealth held in the firm's stock in excess of the market portfolio. Table 1.1 (see Appendix 1.6.3 for all tables) reports the optimal choice of risk by the manager depending on her ambiguity aversion and for different values of risk aversion γ, correlation of the firm's stock with the market ρ, and fraction of wealth θ held in the firm's stock.

Optimal volatility decreases for more risk-averse agents as well as for more ambiguity-averse agents. In the presence of ambiguity, the manager has even stronger incentives not to take on riskier projects and instead to pursue a conservative strategy. If the company-issued stock has a higher market beta (or, equivalently, higher correlation with the market), the manager can take on slightly higher firm-specific volatility. Finally, if the compensation package contains twice as much in stock holdings then the manager chooses half the risk level. This finding is consistent with the notion that the compensation contract should contain additional risk-taking incentives, in particular, stock options. The presence of the

[7] We do not aim for a comprehensive analysis of feasible values that the ambiguity-aversion coefficient could take. A primary goal of this study is a comparative analysis between the reference model with a risk-averse agent and the alternative model with an ambiguity-averse agent. A calibration of ambiguity aversion values to empirical data is a subject for future research.

firm's stock in the managerial portfolio provides a disincentive to implementing riskier projects. The comparative statics are consistent with our intuition about how a manager should behave, and the presence of ambiguity amplifies the incentive to pursue a low-risk strategy regardless of the value of other model parameters.

Second, we illustrate the equilibrium solution of the model. Table 1.2 reports the solution of the optimal contract: the optimal effort u^* and the optimal volatility level σ^* chosen by the manager for given model parameters, the optimal fraction of the manager's wealth in the firm's equity θ^* distributed by shareholders to the manager, and the initial optimal consumption level c_0^*. The results vary across ambiguity-aversion coefficient, risk-aversion coefficient, and correlation of the firm's stock with the market index.

The equilibrium levels of effort and volatility decrease with the manager's ambiguity aversion. Higher risk-aversion also decreases optimal effort, but the volatility profile is either a hump-shaped or decreasing in risk aversion. We observe that ambiguity aversion has a material impact on optimal volatility choice, in particular when compared to a risk-averse manager's choice. If the stock has a higher correlation with the market, it is optimal for the manager to take on higher risk and exert more effort. To compensate for a decrease in the manager's effort and the choice of a lower risk, shareholders should create better incentives and risk-taking incentives for the ambiguity-averse manager. Therefore, shareholders distribute more equity-type compensation to the ambiguity-averse manager in equilibrium. The lower the manager's optimal choice of effort and volatility, the higher the optimal equity compensation given by shareholders. Finally, the manager's initial consumption choice in equilibrium increases with ambiguity aversion and risk aversion.

The next tables show how the ambiguity-averse manager evaluates her compensation plan paid in the firm's stocks or stock options and compare the incentives created by stocks and stock options.

Table 1.3 reports the objective (market) value V_S^M, the ambiguity value V_S^A, and the subjective value V_S^S of the stock held until time T. The ambiguity value V_S^A is the hypothetical expected value of the stock computed under the alternative probability measure. The ambiguity value is considered separately to emphasize the impact of ambiguity aversion. Note that the subjective evaluation includes the manager's ambiguity aversion. The incentive effect of the stock compensation coincides with the stock's subjective value. Table 1.3 also reports stock incentive costs SIC as the ratio of the stock objective value to the subjective value. SIC represents the costs to shareholders of providing incentives to the manager. The results in Table 1.3 vary across different levels of the ambiguity-aversion coefficient and the risk-aversion coefficient.

The ambiguity stock values are lower than the objective values but higher than the initial price. Subjective evaluation driven by the requirement to hold the stock in the managerial portfolio pushes the compensation value to a lower level, below the initial price. However, higher ambiguity aversion slightly increases the subjective evaluation. Therefore, the incentive effect of stocks is higher for the ambiguity-averse manager than for the risk-averse manager. Moreover, the shareholders' costs to provide incentives are lower for the ambiguity-averse manager, as values of SIC decrease with ambiguity aversion.

Table 1.4 reports the objective (market) value V_O^M, the ambiguity value V_O^A, and the subjective value V_O^S of an in-the-money call option, an at-the-money call option and an out-of-the-money call option with strike prices $K \in \{0.8, 1, 1.2\}$ and time to maturity $T = 1$. Similarly to the previous table, the ambiguity value is the option price under the alternative probability measure. Table 1.4 also reports the option deltas computed with respect to the objective value Δ_O^M, the ambiguity value Δ_O^A, and the subjective value Δ_O^S. Option delta measures the incentive effect of the option payout. Table 1.5 reports additional option greeks: the market vega ν^M, the subjective vega ν^S, the market gamma Γ^M, and the subjective gamma Γ^S. Furthermore, Table 1.5 tabulates the option incentive costs OIC computed as the ratio of the option objective value to the option subjective delta.

The market values are higher than both the ambiguity values and the subjective values of the option compensation payout. Ambiguity aversion amplifies the spread between the market value and the subjective value. In-the-money call options are more valuable to the manager than the corresponding out-of-the-money call options. The actual incentives for the manager measured by the subjective delta are lower than shareholders' evaluation of the incentives in terms of market delta. Furthermore, the incentive effect of executive stock options measured by the option delta is lower than the incentive effect of the stock and even a tradable option. However, the option incentive costs OIC are lower than the stock incentive costs SIC regardless of the model parameters. This fact speaks in favor of using stock options rather than stock in the compensation package. Another advantage of options is the risk-taking incentives measured by the option vega. The risk-taking incentives disappear if risk aversion increases. However, the effect is opposite for the ambiguity aversion coefficient. The vega increases with ambiguity aversion in most cases. The option gamma measures the convexity of the compensation payoff, and hence gamma also helps offsetting the risk-reducing behavior of managers. As is the case with vega, option gamma increases with ambiguity aversion. Note that the market values of both vega and gamma are higher than their subjective values.

Relying on the comparison of subjective values and incentive effects, we conclude that an ambiguity-averse manager requires higher effort incentives and risk-taking incentives in her compensation plan. Both stocks and stock options provide incentives. While stocks provide stronger incentives, the costs to shareholders of providing incentives are lower for option compensation. Moreover, risk-taking incentives created by executive stock options are of paramount importance for the ambiguity-averse manager.

1.5 Conclusion

This paper explains the prevalence of stock options in executive compensation plans by incorporating the manager ambiguity aversion in the optimal contract design. We develop a principal-agent model of the optimal compensation plan in which the manager chooses a level of effort and idiosyncratic volatility. The manager is constrained to hold a certain share of her wealth in the company's stock.

We find a significant negative effect of ambiguity aversion on both the optimal effort level and the optimal risk selected by the manager. The model predicts a higher level of equity-based compensation to offset the loss of effort incentives and risk-taking incentives caused by ambiguity aversion. The paper compares the values of the stock and stock option compensation payouts from the manager's and shareholders' points of view. The manager's ambiguity aversion amplifies the gap between the subjective value and the market value of stock-based compensation. Both stocks and stock options create incentives that are particularly important for ambiguity-averse managers. However, option compensation has two main advantages: lower costs to shareholders and stronger risk-taking incentives.

There are two main directions for future research. An empirical study of this model would provide us with reliable quantitative estimates of executive stock option holdings in actual compensation contracts and thus allow us to calibrate the ambiguity-aversion coefficient. Moreover, we could compare the structure of compensation contracts in periods of high and low levels of ambiguity in the economy and assess the efficiency of executive stock options relative to alternative types of compensation.

1.6 Appendices

1.6.1 Optimal Solution to the Shareholders' and Manager's Optimization Problem

This section contains a detailed solution of the model presented in Section 1.3.

Assume B_t follows a deterministic process

$$dB_t = rB_t dt, \tag{1.39}$$

M_t follows the $\mathcal{P}$-dynamics

$$\frac{dM_t}{M_t} = \mu_M dt + \sigma_M dZ_t^M, \tag{1.40}$$

and S_t follows the $\mathcal{P}$-dynamics

$$\frac{dS_t}{S_t} = \mu dt + \sigma\sqrt{1-\rho^2}dZ_t^S + \sigma\rho dZ_t^M, \tag{1.41}$$

where

$$\mu = \xi u + \eta\sigma + r + \frac{\sigma\rho}{\sigma_M}(\mu_M - r),\ \xi > 0, \eta > 0,$$

and

$$\mathbb{E}(dZ_t^S dZ_t^M) = \rho dt,\ \rho \in [-1, 1].$$

Denote the set of alternative probability measures by $\mathcal{Q}^h$. Let $\mathcal{Q}^h$ be absolutely continuous with respect to the reference probability measure $\mathcal{P}$. The Radon-Nikodym derivative (or density) $\frac{d\mathcal{Q}^h}{d\mathcal{P}}$ coincides with its conditional expectation, and hence is a martingale. Under Novikov's Condition, the density is an exponential martingale that is equal to the following:

$$\frac{d\mathcal{Q}^h}{d\mathcal{P}} = \exp\{-\int_0^T \frac{||h_t||^2}{2}dt - \int_0^T h_t dZ_t\}, \tag{1.42}$$

where $Z_t = (Z_t^S, Z_t^M)$ is a vector of $\mathcal{P}$-Brownian motions and $h_t = (h_t^S, h_t^M)$ is an adapted process such that $Z_t^h = Z_t + \int_0^t h_s ds$ is a $\mathcal{Q}^h$-Brownian motion. Existence of the process h_t is determined by Girsanov's Theorem. In our framework, h_t denotes a vector of ambiguity parameters related to the corresponding source of risk. The distorted $\mathcal{Q}^h$-dynamics of assets in the economy are obtained by substituting the Z_t^h-Brownian motion for the Z_t-Brownian motion. Computations show that the model's misspecification is determined only by the drift change of the corresponding process:

$$\frac{dM_t}{M_t} = (\mu_M + \sigma_M h_t^M)dt + \sigma_M dZ_t^M \tag{1.43}$$

and

$$\frac{dS_t}{S_t} = (\mu + \sigma(\rho h_t^M + \sqrt{1-\rho^2} h_t^S))dt + \sigma(\rho dZ_t^M + \sqrt{1-\rho^2} dZ_t^S). \tag{1.44}$$

Let θ denote the fraction of the manager's wealth invested in the firm's stock in excess of the market portfolio, θ_M denote the fraction of wealth invested in the market index (the market portfolio), and $1 - \theta - \theta_M$ denote the fraction of wealth invested in the risk-free bond. Assume the manager is endowed with positive initial wealth $W_0 > 0$. Current wealth W_t has the following $\mathcal{P}$-dynamics:

$$\begin{aligned} dW_t &= \left(\left(\theta_M(\mu^M - r) + \theta(\mu - r) + r\right) W_t - C_t\right) dt \\ &\quad + \left((\theta_M \sigma_M + \theta\sigma\rho)dZ_t^M + \theta\sigma\sqrt{1-\rho^2} dZ_t\right) W_t. \end{aligned} \tag{1.45}$$

The $\mathcal{Q}^h$-dynamics of wealth are the following:

$$\begin{aligned} dW_t &= \left(\left(\theta_M(\mu_M - r + \sigma_M h_t^M) + \theta(\mu - r + \sigma(\rho + \sqrt{1-\rho^2})h_t^S) + r\right) W_t - C_t\right) dt \\ &\quad + \left((\theta_M \sigma_M + \theta\sigma\rho)dZ_t^M + \theta\sigma\sqrt{1-\rho^2} dZ_t\right) W_t. \end{aligned} \tag{1.46}$$

The manager's optimization problem subject to the $\mathcal{Q}^h$-dynamics of wealth is the following:

$$\max_{u,\sigma}\max_{\theta_M,C_t}\min_{h_t}\mathbb{E}^h\int_0^\infty e^{-rt}\left(\frac{C_t^{1-\gamma}}{1-\gamma}-\frac{u_t^2}{2}+\psi\frac{h_t^2}{2}\right)dt, \tag{1.47}$$

where $\gamma > 1$ is the manager's relative risk-aversion coefficient[8], the coefficient $\psi \geq 0$ is inversely proportional to the manager's ambiguity aversion, consumption is nonnegative, $C_t \geq 0$, and initial wealth is positive, $W_0 > 0$.

The shareholder sets the optimal equity compensation θ in order to maximize the expected stock price at given compensation horizon T:

$$\max_{\theta}\mathbb{E}S_T. \tag{1.48}$$

In order to determine the optimal choice of investment, consumption, volatility, and managerial effort, we rely on the stochastic dynamic programming technique that leads to the HJB optimality equation.

Solving the optimization problem for the manager requires three steps. First, we solve the minimization part for the optimal ambiguity parameter h_t. Second, we solve the HJB equation for the optimal consumption and investment as in Merton (1969) and, simultaneously, for the optimal effort and idiosyncratic volatility. Note that the minimization cannot be interchanged with the maximizations because doing so would lead to a different solution.

We define the indirect utility function (or value function) as follows:

$$J(W(s),s)=\max_{u,\sigma}\max_{\theta_M,C_t}\min_{h_t}\mathbb{E}^h\int_s^\infty e^{-r(t-s)}\left(\frac{C_t^{1-\gamma}}{1-\gamma}-\frac{u^2}{2}+\psi\frac{h_t^2}{2}\right)dt. \tag{1.49}$$

The indirect utility function is independent of explicit time due to the infinite time horizon:

$$J(W(s),s)=J(W(0),0)=J. \tag{1.50}$$

[8]We do not consider the case $\gamma = 1$, which would correspond to logarithmic utility.

Hence the optimality HJB equation is an ordinary differential equation for J:

$$\begin{aligned}
0 \;=\; & \max_{u,\sigma} \max_{\theta_M, C_t} \min_{h_t^S, h_t^M} \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{u^2}{2} + \psi \frac{(h_t^S)^2 + (h_t^M)^2}{2} - rJ \\
& + J_W \left(\left(\theta_M (\mu_M - r + \sigma_M h_t^M) + \theta(\mu - r + \sigma(\rho + \sqrt{1-\rho^2}) h_t^S) + r \right) W_t - C_t \right) \\
& + 0.5 J_{WW} W^2 \left((\theta_M \sigma_M + \theta \sigma \rho)^2 + \theta^2 \sigma^2 (1-\rho^2) \right), \qquad (1.51)
\end{aligned}$$

subject to the boundary (transversality) condition $\lim_{t\to\infty} \mathbb{E}^h e^{-rt} J(W(t)) = 0$.

Following Merton (1969), conjecture an indirect utility function J of the form:

$$J(W) = \frac{a^{-\gamma}}{1-\gamma} W^{1-\gamma}, \qquad (1.52)$$

where a is a constant in the infinite-horizon case.

Note that $-J_{WW}W/J_W = \gamma$ is the relative risk-aversion coefficient. Moreover, we use the specification of the penalty function proposed by Maenhout (2004):

$$\psi = \frac{J(1-\gamma)}{\Omega} = \frac{J_W W}{\Omega}, \qquad (1.53)$$

where Ω is a constant that reflects ambiguity aversion.

The solution to the minimization problem exists due to the convexity of the objective function with respect to h_t^S and h_t^M. The first-order condition is sufficient and yields the following:

$$h_t^M = -\frac{J_W W}{\psi} \theta_M \sigma_M \text{ and } h_t^S = -\frac{J_W W}{\psi} \theta \sigma (\rho + \sqrt{1-\rho^2}), \qquad (1.54)$$

or, using (1.53),

$$h_t^M = -\Omega \theta_M \sigma_M \text{ and } h_t^S = -\Omega \theta \sigma (\rho + \sqrt{1-\rho^2}). \qquad (1.55)$$

We substitute these values of h_t^M and h_t^S back into the HJB equation:

$$\begin{aligned} 0 &= \max_{u,\sigma} \max_{\theta_M, C_t} -\frac{\Omega J_W W}{2}\left(\theta_M^2\sigma_M^2 + \theta^2\sigma^2(\rho + \sqrt{1-\rho^2})^2\right) \\ &\quad + \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{u^2}{2} - rJ + J_W\left((\theta_M(\mu_M - r) + \theta(\mu - r) + r)W_t - C_t\right) \\ &\quad + 0.5 J_{WW} W^2\left((\theta_M\sigma_M + \theta\sigma\rho)^2 + \theta^2\sigma^2(1-\rho^2)\right). \end{aligned} \tag{1.56}$$

Next, we derive the first-order conditions for the maximization part. First, optimal consumption is equal to

$$C_t^* = (J_W)^{-1/\gamma}. \tag{1.57}$$

Consumption depends on wealth as follows:

$$C_t^* = aW_t. \tag{1.58}$$

Second, the optimal investment in the index is a Merton's portfolio adjusted for ambiguity aversion and excess holding of the firm's stock:

$$\theta_M^* = \frac{\mu_M - r - \gamma\theta\sigma\rho\sigma_M}{\sigma_M^2(\gamma + \Omega)}. \tag{1.59}$$

Since the market beta is $\beta = \frac{\sigma\rho}{\sigma_M}$, we obtain the optimal fraction of the wealth invested in the market:

$$\theta_M^* = \frac{\mu_M - r - \gamma\theta\beta\sigma_M^2}{\sigma_M^2(\gamma + \Omega)}. \tag{1.60}$$

Third, the solution to the maximization problem with respect to the variables of managerial choice (effort and volatility) exists due to concavity of the quadratic objective function in u and σ. The optimal level of effort is the following:

$$u^* = \theta\xi J_W W_t = \theta\xi a^{-\gamma} W_t^{1-\gamma}. \tag{1.61}$$

We consider a constant level of effort in the model. Hence, optimal effort depends on initial wealth:

$$u^* = \theta\xi a^{-\gamma} W_0^{1-\gamma}. \tag{1.62}$$

Finally, optimal firm-specific volatility is the following:

$$\sigma^* = \frac{\eta + \frac{\rho(\mu_M - r)}{\sigma_M} - \gamma\rho\sigma_M\theta_M}{\theta(\gamma + \Omega(\rho + \sqrt{1-\rho^2})^2)}. \tag{1.63}$$

If we substitute (1.59) for θ_M in (1.63), we obtain the following explicit formula for the optimal volatility:

$$\sigma^* = \frac{\eta + \frac{\rho\Omega(\mu_M - r)}{\sigma_M(\Omega+\gamma)}}{\theta\left(\gamma + \Omega(\rho + \sqrt{1-\rho^2})^2\right)\left(1 - \frac{\gamma^2\rho^2}{(\gamma+\Omega)(\gamma+\Omega(\rho+\sqrt{1-\rho^2})^2)}\right)}. \tag{1.64}$$

We denote $\sigma^*\theta = x$, where $x = x(\Omega)$ is a function independent of explicit variables chosen by the manager and shareholders such as C_t, θ_M, and θ. $x(\Omega)$ is decreasing in Ω and $x(0) = \frac{\eta}{\gamma(1-\rho^2)}$. Using $x = x(\Omega)$, the optimal fraction of wealth invested in the market index is the following:

$$\theta_M^* = \frac{\mu_M - r}{\sigma_M^2(\gamma+\Omega)} - \frac{\gamma\rho x}{\sigma_M(\gamma+\Omega)}. \tag{1.65}$$

The constant a in the indirect utility function (1.52) is computed by substituting (1.52) for J in (1.51) and using the optimal values C_t^* and u^*:

$$\begin{aligned} 0 \;=\; & \frac{a^{1-\gamma}W^{1-\gamma}}{1-\gamma} - \frac{1}{2}\theta^2\xi^2 a^{-2\gamma}W^{2-2\gamma} + \Omega a^{-\gamma}W^{1-\gamma}\left(\theta_M^2\sigma_M^2 + \theta^2\sigma^2(\rho+\sqrt{1-\rho^2})^2\right) \\ & + a^{-\gamma}W^{1-\gamma}\left(\theta_M(\mu_M - r - \Omega\theta_M\sigma_M^2) + \theta(\mu - r - \Omega\theta\sigma^2(\rho+\sqrt{1-\rho^2})^2 + r - a\right) \\ & - \frac{ra^{-\gamma}W^{1-\gamma}}{1-\gamma} - \gamma^{-\gamma}W^{1-\gamma}\left((\theta_M\sigma_M + \theta\sigma\rho)^2 + \theta^2\sigma^2(1-\rho^2)\right). \end{aligned} \tag{1.66}$$

The solution of the following equation yields the value of a:

$$\begin{aligned} a &= r + \frac{\gamma - 1}{\gamma}(\frac{1}{2}\theta^2\xi^2 W_0^{1-\gamma}a^{-\gamma} + \theta_M^*(\mu_M - r) + \theta(\mu - r) \\ &\quad -\gamma((\theta_M^*\sigma_M + \theta\sigma^*\rho)^2 + \theta^2(\sigma^*)^2(1-\rho^2))). \end{aligned} \tag{1.67}$$

Using the optimal values θ_M^*, and σ^*, we obtain the following equation:

$$\begin{aligned} 0 &= \frac{\gamma}{1-\gamma}(a - r) + \frac{1}{2}\theta^2\xi^2 a^{-\gamma}W_0^{1-\gamma} + \frac{(\mu_M - r)^2\Omega}{\sigma_M^2(\gamma + \Omega)} + \frac{2\gamma(\gamma - 1)(\mu_M - r)}{\gamma + \Omega} \\ &\quad + \left(\eta + \frac{\Omega\rho(\mu_M - r)}{\sigma_M(\gamma + \Omega)}\right)x + \left(\frac{\gamma^2(\gamma + 2\Omega)\rho^2}{(\gamma + \Omega)^2} - \gamma^2\right)x^2. \end{aligned} \tag{1.68}$$

We denote the terms independent of θ by y, and the equation for a becomes:

$$a = r + \frac{\gamma - 1}{2\gamma}\theta^2\xi^2 a^{-\gamma}W_0^{1-\gamma} + y, \tag{1.69}$$

where y depends only on risk and ambiguity preferences, the market risk premium, the risk-free rate (discount rate), and the correlation coefficient ρ. y decreases in Ω and so does a. In the absence of effort, $\xi = 0$, the solution for a does not depend on θ:

$$a = r + y. \tag{1.70}$$

Taking the expected value of S_T and substituting the optimal values of effort u^* and volatility σ^*, the principal's problem becomes:

$$\max_\theta \mathbb{E}S_T = \max_\theta S_0 e^{(r + \frac{\sigma^*\rho}{\sigma_M}(\mu_M - r) + \xi u^* + \eta\sigma^*)T}. \tag{1.71}$$

The optimal θ^* is found as a solution to the following implicit equation:

$$\theta^*\xi^2 a^{-\gamma}W_0^{1-\gamma} = x\left(\eta + \frac{\rho(\mu_M - r)}{\sigma_M}\right). \tag{1.72}$$

1.6.2 Subjective and Objective Evaluation of Equity Compensation

This section presents details of the equity compensation evaluations. We evaluate the equity compensation paid by either shares of stock or stock options, from the shareholders' point of view (objective evaluation) and from the manager's point of view (subjective evaluation).

The objective evaluation of the equity compensation coincides with its market value computed as the expected payoff of stocks and stock options at time T, where expectations are taken under the reference probability measure. However, the manager's optimal choice of effort u^* and volatility σ^* still affects the shareholders' objective evaluation via the mispricing term α in equation (1.5):

$$\alpha = \xi u + \eta\sigma. \tag{1.73}$$

The mispricing α increases the expected stock return μ as determined by equation (1.4):

$$\mu = r + \alpha + \frac{\sigma\rho}{\sigma_M}(\mu_M - r). \tag{1.74}$$

The objective evaluation of the stock held until time T is given by the following formula:

$$V_S^M = \mathbb{E}S_T = S_0 e^{\mu T}. \tag{1.75}$$

The objective evaluation of an executive stock option is given by the Black-Scholes value, and the mispricing term α plays the role of a dividend yield in the Black-Scholes model:

$$V_O^M = S_0 e^{\alpha T}\mathcal{N}(d_1) - Ke^{-rT}\mathcal{N}(d_2), \tag{1.76}$$

where T is the time to maturity, K is the strike price, and $d_{1,2} = \frac{\ln(S_0/K)+(r+\alpha\pm\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$.

The incentive effect of option-based compensation is measured by the option delta, the sensitivity of the option value with respect to the underlying stock's price:

$$\Delta_O^M = e^{\alpha T}\mathcal{N}(d_1). \tag{1.77}$$

The convexity of compensation payoff is given by the option gamma, the sensitivity of the option delta with respect to the underlying stock's price:

$$\Gamma^M = \frac{e^{\alpha T}}{S_0\sigma\sqrt{T}}\frac{\partial\mathcal{N}(d_1)}{\partial d_1}. \tag{1.78}$$

The risk-taking incentive is given by the option vega, the sensitivity of the option value to volatility:

$$\nu^M = S_0\sqrt{T}e^{\alpha T}\frac{\partial\mathcal{N}(d_1)}{\partial d_1} - \eta e^{\alpha T}\mathcal{N}(d_1). \tag{1.79}$$

Note that the equations for delta and gamma are standard formulas for the greeks in the Black-Scholes model for a dividend-paying stock. However, the formula for vega requires an adjustment for the mispricing α dependent on σ.

The manager's subjective evaluation of the compensation contract is based on two arguments. First, the subjective evaluation is driven by manager's ambiguity aversion. As shown in equation (1.9), ambiguity distortion affects the dynamics of stock returns via the mispricing term α. The distorted α^h is given by:

$$\alpha^h = \xi u + \eta\sigma + \sigma(\rho h_t^M + \sqrt{1-\rho^2}h_t^S). \tag{1.80}$$

Substitute optimal values of h_t^M and h_t^S from equation (1.55):

$$\alpha^h = \xi u + \eta\sigma - \sigma\Omega(\rho\sigma_M\theta_M + \theta\sqrt{1-\rho^2}(\rho + \sqrt{1-\rho^2})). \tag{1.81}$$

Thus, the expected stock return for the ambiguity-averse manager equals the following:

$$\mu^h = \xi u + \eta\sigma - \sigma\Omega(\rho\sigma_M\theta_M + \theta\sqrt{1-\rho^2}(\rho + \sqrt{1-\rho^2})) + r + \frac{\sigma\rho}{\sigma_M}(\mu_M - r). \tag{1.82}$$

Note that ambiguity aversion affects μ not only via the explicit additional term α^h but also implicitly via the equilibrium values of σ^*, u^*, and θ^*. The evaluation of the equity compensation by the ambiguity-averse manager is equivalent to its market evaluation with the distorted stock dynamics μ^h under the alternative probability measure. The expected payoff of stocks held until time T for the ambiguity-averse manager is the following:

$$V_S^A = \mathbb{E}^h S_T = S_0 e^{\mu^h T}. \tag{1.83}$$

The evaluation of stock options for the ambiguity-averse manager is given by the Black-Scholes value with mispricing term α^h:

$$V_O^A = S_0 e^{\alpha^h T}\mathcal{N}(d_1^h) - Ke^{-rT}\mathcal{N}(d_2^h), \tag{1.84}$$

where T is the time to maturity, K is the strike price, and $d_{1,2}^h = \frac{\ln(S_0/K)+(r+\alpha^h \pm \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$. The option greeks are also evaluated using α^h:

$$\Delta_O^A = e^{\alpha^h T}\mathcal{N}(d_1^h), \tag{1.85}$$

$$\Gamma^A = \frac{e^{\alpha^h T}}{S_0\sigma\sqrt{T}}\frac{\partial\mathcal{N}(d_1^h)}{\partial d_1^h}, \tag{1.86}$$

$$\begin{aligned}\nu^A &= -e^{\alpha^h T}\mathcal{N}(d_1^h)\left(\eta - \Omega(\rho\sigma_M\theta_M + \sqrt{1-\rho^2}(\rho + \sqrt{1-\rho^2})\theta)\right) \\ &\quad + S_0\sqrt{T}e^{\alpha^h T}\frac{\partial\mathcal{N}(d_1^h)}{\partial d_1^h}. \end{aligned} \tag{1.87}$$

The second argument in the subjective evaluation of equity-based compensation is the fact that the manager holds an undiversified portfolio with an overweighted position in the firm's equity due to compensation contract obligations. The subjective evaluation is based on Ingersoll (2006), who computes a subjective asset value for an undiversified (restricted to hold the firm's stock until time T) manager using her marginal utility as a martingale pricing process. This same principle is applicable in our framework.

Subjective evaluations affect both the interest rate and the risky cash flows; in other words, the dividend yield. Details of the derivation can be found in Ingersoll (2006). According to Ingersoll (2006), the subjective interest rate $\tilde{r}$ is lower than the market interest rate:

$$\tilde{r} = r - \gamma\theta^2\sigma^2(1-\rho^2). \tag{1.88}$$

The subjective dividend yield $\tilde{q}$ is higher than the actual dividend yield, or, in our setting, than the mispricing term α^h (1.81):

$$\tilde{q} = -\alpha^h + \gamma(1-\theta)\theta\sigma^2(1-\rho^2). \tag{1.89}$$

Equivalently, the subjective expected stock return or discount rate $\tilde{\mu}$ is higher than the actual expected stock return μ^h by the same adjustment term:

$$\tilde{\mu} - \mu^h = \tilde{q} + \alpha^h = \gamma(1-\theta)\theta\sigma^2(1-\rho^2). \tag{1.90}$$

The manager discounts future cash flows at a higher rate $\tilde{\mu}$ than the rate μ^h for the ambiguity-averse manager. However, μ^h is lower than the market rate μ.

Stocks and stock options in the compensation contract are valued by the manager at the optimal values of σ^* and θ^* with an adjusted interest rate $\tilde{r}$ and an adjusted dividend yield $\tilde{q}$. As shown in Ingersoll (2006), the subjective value of a restricted stock held until time T equals the following:

$$V_S^S = e^{-T\tilde{\mu}}\mathbb{E}^h S_T = e^{(\mu^h - \tilde{\mu})T} S_0 = e^{-(\gamma(1-\theta)\theta\sigma^2(1-\rho^2))T} S_0. \tag{1.91}$$

The incentive effect of stock compensation or the stock subjective delta is given by the sensitivity of the subjective stock value to the stock price:

$$\Delta_S = e^{-(\gamma(1-\theta)\theta\sigma^2(1-\rho^2))T}. \tag{1.92}$$

The risk-neutral drift in the option pricing equation is affected by the subjective adjustment:

$$\tilde{r} - \tilde{q} = r + \alpha^h - \gamma\theta\sigma^2(1-\rho^2). \tag{1.93}$$

Ingersoll (2006) derives the subjective option value as a Black-Scholes value with an adjusted interest rate $\tilde{r}$ and an adjusted dividend yield $\tilde{q}$:

$$V_O^S = S_0 e^{-T\tilde{q}}\mathcal{N}(\widetilde{d}_1) - Ke^{-T\tilde{r}}\mathcal{N}(\widetilde{d}_2), \tag{1.94}$$

where T is the time to maturity, K is the strike price, and $\widetilde{d}_{1,2} = \frac{\ln(S_0/K)+(\tilde{r}-\tilde{q}\pm\frac{\sigma^2}{2}T)}{\sigma\sqrt{T}}$.

The option greeks are evaluated using similar adjustments. The subjective option delta is

$$\Delta_O^S = e^{-T\tilde{q}}\mathcal{N}(\tilde{d}_1). \tag{1.95}$$

The subjective option gamma is the following:

$$\Gamma^S = \frac{e^{-T\tilde{q}}}{S_0\sigma\sqrt{T}}\frac{\partial\mathcal{N}(\widetilde{d}_1)}{\partial\widetilde{d}_1}. \tag{1.96}$$

The subjective option vega is the following:

$$\nu^S = -e^{\alpha^h T}\mathcal{N}(d_1^h)\left(\eta - \Omega\left(\rho\sigma_M\theta_M + \sqrt{1-\rho^2}(\rho + \sqrt{1-\rho^2})\theta\right) - 2\gamma\theta\sigma(1-\rho^2)\right) + S_0\sqrt{T}e^{\alpha^h T}\frac{\partial\mathcal{N}(d_1^h)}{\partial d_1^h}. \tag{1.97}$$

Finally, we measure the costs to a shareholder of providing incentives as a market compensation value per unit of subjective delta. The stock incentive costs are computed as follows:

$$SIC = \frac{V_S^M}{\Delta_S}, \tag{1.98}$$

and the option incentive costs equal the following:

$$OIC = \frac{V_O^M}{\Delta_O^S}. \tag{1.99}$$

1.6.3 Tables

Table 1.1: Comparative statics of the manager's volatility choice σ. The fixed parameters are the following: risk-free rate $r = 4\%$, expected rate of market return $\mu_M = 10\%$, market volatility $\sigma_M = 0.3$, growth prospects $\eta = 0.5$. We vary the following parameters: manager's holdings of the firm's stock $\theta \in \{0.3, 0.6\}$, relative risk-aversion coefficient $\gamma \in \{2, 3, 4\}$, correlation coefficient between the stock return and the market return $\rho \in \{0.1, 0.4\}$, and ambiguity-aversion coefficient $\Omega \in \{0.01, 0.5, 1\}$.

Correlation			$\rho = 0.1$			
Equity holdings		$\theta = 0.3$			$\theta = 0.6$	
$\gamma =$	2	3	4	2	3	4
Ω			**Risk Choice** σ			
0.01	0.84	0.56	0.42	0.42	0.28	0.21
0.5	0.65	0.47	0.37	0.33	0.23	0.18
1	0.53	0.40	0.33	0.27	0.20	0.16

Correlation			$\rho = 0.4$			
Equity holdings		$\theta = 0.3$			$\theta = 0.6$	
$\gamma =$	2	3	4	2	3	4
Ω			**Risk Choice** σ			
0.01	0.98	0.66	0.49	0.49	0.33	0.25
0.5	0.66	0.49	0.39	0.33	0.25	0.20
1	0.50	0.40	0.33	0.25	0.20	0.16

Table 1.2: Comparative statics of the equilibrium solution. The table reports the manager's optimal effort u^*, optimal volatility σ^*, the optimal equity compensation (i.e., the fraction θ^* of wealth invested in the firm's stock in excess of the market portfolio), and the initial optimal consumption c_0^*. The fixed parameters are the following: risk-free rate $r = 4\%$, expected rate of market return $\mu_M = 10\%$, market volatility $\sigma_M = 0.3$, initial value of the firm's stock price $S_0 = 1$, initial wealth of the manager $W_0 = 1$, growth prospects $\eta = 0.5$, and effort multiplier $\xi = 0.5$. We vary the following parameters: relative risk-aversion coefficient $\gamma \in \{2, 3, 4\}$, correlation coefficient between the stock return and the market return $\rho \in \{0.1, 0.4\}$, and ambiguity-aversion coefficient $\Omega \in \{0.01, 0.5, 1\}$.

	$\rho = 0.1$			$\rho = 0.4$		
$\gamma =$	2	3	4	2	3	4
Ω	**Effort** u^*					
0.01	31.16	20.81	15.62	57.96	38.72	29.07
0.5	25.15	17.94	13.95	44.75	32.25	25.24
1	21.02	15.74	12.58	36.75	27.79	22.37
Ω	**Volatility** σ^*					
0.01	2.92	6.06	5.19	13.25	28.46	15.93
0.5	0.94	3.05	3.86	0.51	2.09	3.29
1	0.69	2.34	3.44	0.36	1.24	2.06
Ω	**Equity Compensation** θ^*					
0.01	0.09	0.03	0.02	0.02	0.01	0.01
0.5	0.22	0.05	0.03	0.45	0.08	0.04
1	0.25	0.05	0.03	0.53	0.11	0.06
Ω	**Consumption** c_0^*					
0.01	0.04	0.09	0.17	0.01	0.04	0.11
0.5	0.07	0.11	0.18	0.07	0.11	0.17
1	0.08	0.12	0.18	0.08	0.13	0.19

Table 1.3: Stock compensation. The table reports the following values predicted by the equilibrium solution of the model: stock market value V_S^M, stock ambiguity value V_S^A, stock subjective value V_S^S, and stock incentive costs SIC. The fixed parameters are the following: risk-free rate $r = 4\%$, expected rate of market return $\mu_M = 10\%$, market volatility $\sigma_M = 0.3$, initial value of the firm's stock price $S_0 = 1$, initial wealth of the manager $W_0 = 1$, time horizon $T = 1$, correlation coefficient between the stock return and the market return $\rho = 0.1$, growth prospects $\eta = 0.5$, and effort multiplier $\xi = 0.5$. We vary the following parameters: relative risk-aversion coefficient $\gamma \in \{2, 3, 4\}$ and ambiguity-aversion coefficient $\Omega \in \{0.01, 0.5, 1\}$.

$\gamma =$	2	3	4
Ω		V_S^M	
0.01	1.75	2.46	2.15
0.5	1.33	1.67	1.81
1	1.26	1.51	1.70
Ω		V_S^A	
0.01	1.74	2.45	2.14
0.5	1.16	1.46	1.61
1	1.02	1.22	1.40
Ω		V_S^S	
0.01	0.27	0.05	0.08
0.5	0.74	0.29	0.19
1	0.84	0.43	0.26
Ω		SIC	
0.01	6.60	46.38	27.99
0.5	1.79	5.81	9.57
1	1.50	3.47	6.52

Table 1.4: Option compensation. The table reports the following values predicted by the equilibrium solution of the model: option market value V_O^M, option ambiguity value V_O^A, option subjective value V_O^S, delta of the option market value Δ_O^M, delta of the option ambiguity value Δ_O^A, and delta of the option subjective value Δ_O^S. The fixed parameters are the following: risk-free rate $r = 4\%$, expected rate of market return $\mu_M = 10\%$, market volatility $\sigma_M = 0.3$, initial value of the firm's stock price $S_0 = 1$, initial wealth of the manager $W_0 = 1$, time to maturity $T = 1$, correlation coefficient between the stock return and the market return $\rho = 0.1$, growth prospects $\eta = 0.5$, and effort multiplier $\xi = 0.5$. We vary the following parameters: relative risk-aversion coefficient $\gamma \in \{2, 3, 4\}$, ambiguity-aversion coefficient $\Omega \in \{0.01, 0.5, 1\}$, and option strike price $K \in \{0.8, 1, 1.2\}$.

$K =$		0.8			1			1.2	
$\gamma =$	2	3	4	2	3	4	2	3	4
Ω					V_O^M				
0.01	0.81	0.80	0.78	0.81	0.81	0.78	0.81	0.81	0.78
0.5	0.47	0.76	0.77	0.43	0.76	0.77	0.40	0.76	0.77
1	0.37	0.71	0.76	0.33	0.70	0.76	0.29	0.70	0.75
Ω					V_O^A				
0.01	0.81	0.80	0.78	0.81	0.81	0.78	0.80	0.81	0.78
0.5	0.39	0.67	0.69	0.35	0.67	0.69	0.32	0.67	0.69
1	0.27	0.57	0.62	0.23	0.56	0.62	0.18	0.55	0.62
Ω					V_O^S				
0.01	0.20	0.04	0.06	0.20	0.04	0.06	0.20	0.05	0.06
0.5	0.24	0.19	0.13	0.21	0.18	0.13	0.17	0.18	0.13
1	0.19	0.23	0.16	0.14	0.23	0.16	0.10	0.22	0.16
Ω					Δ_O^M				
0.01	0.83	0.81	0.78	0.83	0.81	0.78	0.82	0.81	0.78
0.5	0.72	0.78	0.77	0.66	0.78	0.77	0.60	0.77	0.77
1	0.70	0.75	0.76	0.62	0.74	0.76	0.52	0.73	0.76
Ω					Δ_O^A				
0.01	0.82	0.81	0.78	0.82	0.81	0.78	0.82	0.81	0.78
0.5	0.60	0.68	0.69	0.54	0.68	0.69	0.48	0.68	0.69
1	0.50	0.60	0.63	0.41	0.60	0.62	0.31	0.59	0.63
Ω					Δ_O^S				
0.01	0.21	0.04	0.06	0.21	0.04	0.06	0.20	0.04	0.06
0.5	0.36	0.19	0.13	0.29	0.18	0.13	0.24	0.19	0.13
1	0.34	0.24	0.16	0.24	0.23	0.16	0.17	0.23	0.16

Table 1.5: Option compensation (continued). The table reports the following values predicted by the equilibrium solution of the model: option incentive costs OIC, option vega market value ν^M, option vega subjective value ν^S, option gamma market value Γ^M, and option gamma subjective value Γ^S. The fixed parameters are the following: risk-free rate $r = 4\%$, expected rate of market return $\mu_M = 10\%$, market volatility $\sigma_M = 0.3$, initial value of the firm's stock price $S_0 = 1$, initial wealth of the manager $W_0 = 1$, time to maturity $T = 1$, correlation coefficient between the stock return and the market return $\rho = 0.1$, growth prospects $\eta = 0.5$, and effort multiplier $\xi = 0.5$. We vary the following parameters: relative risk-aversion coefficient $\gamma \in \{2, 3, 4\}$, ambiguity-aversion coefficient $\Omega \in \{0.01, 0.5, 1\}$, and option strike price $K \in \{0.8, 1, 1.2\}$.

$K =$		0.8			1			1.2	
$\gamma =$	2	3	4	2	3	4	2	3	4
Ω					OIC				
0.01	3.88	18.91	12.58	3.93	18.91	12.58	3.98	18.91	12.58
0.5	1.29	4.06	5.97	1.46	4.10	5.99	1.67	4.14	6.00
1	1.10	2.92	4.70	1.35	2.99	4.72	1.71	3.05	4.74
Ω					ν^M				
0.01	0.13	0.00	0.01	0.15	0.00	0.02	0.17	0.01	0.02
0.5	0.31	0.12	0.06	0.37	0.13	0.07	0.41	0.15	0.07
1	0.30	0.18	0.08	0.38	0.21	0.09	0.43	0.23	0.10
Ω					ν^S				
0.01	0.07	0.01	0.01	0.07	0.00	0.01	0.08	0.00	0.01
0.5	0.23	0.05	0.02	0.25	0.05	0.02	0.26	0.06	0.02
1	0.19	0.09	0.03	0.23	0.10	0.03	0.23	0.10	0.03
Ω					Γ^M				
0.01	0.04	0.00	0.00	0.05	0.00	0.00	0.05	0.00	0.00
0.5	0.32	0.04	0.01	0.39	0.04	0.01	0.43	0.05	0.02
1	0.44	0.08	0.02	0.55	0.08	0.03	0.62	0.09	0.03
Ω					Γ^S				
0.01	0.02	0.00	0.00	0.02	0.00	0.00	0.03	0.00	0.00
0.5	0.29	0.02	0.00	0.31	0.02	0.00	0.31	0.02	0.01
1	0.42	0.04	0.01	0.44	0.05	0.01	0.41	0.05	0.01

Chapter 2

Long/Short Equity Hedge Funds and Systematic Ambiguity

Abstract

This paper presents a hedge fund portfolio choice model for an investor facing ambiguity. In the empirical section, we measure ambiguity as the cross-sectional dispersion in forecasts of industrial production growth and stock market returns, and we construct systematic ambiguity factors from the universe of S&P 500 stocks. We estimate ambiguity betas for long/short equity hedge fund strategies and document significant ambiguity exposures for directional long/short equity hedge funds. We compare the out-of-sample performance of portfolios constructed according to hedge fund alphas with and without systematic ambiguity exposures and find that the former outperform the latter.[1]

JEL codes: G11

Keywords: Ambiguity, Asset Allocation, Long/Short Equity Hedge Funds, Performance Measurement.

[1]This chapter is a co-authored paper with Professor Rajna Gibson Brandon.

2.1 Introduction

What is ambiguity? Investors act under ambiguity when they do not know the exact probability measure associated with external events that may influence a decision or a choice. Ambiguity differs from the concept of risk because it focuses on probabilistic uncertainty rather than uncertainty with respect to the possible realizations of an event. Knight (1921) was the first to emphasize the importance of ambiguity for economic decisions. Economic agents behave differently when they know the probability distribution for uncertain outcomes (known unknowns) than when they do not know it and thus act under ambiguity (unknown unknowns). This finding is supported by psychological experiments such as the Ellsberg Paradox (1961) that demonstrates that people are more likely to take on more risk when the event probabilities are known.[2] Such experiments demonstrated the existence of ambiguity aversion.

The notion of ambiguity should be relevant for hedge fund investors due to various sources of ambiguity associated with hedge fund investments. First, the opacity of hedge fund strategies may generate ambiguity with respect to hedge funds' risk exposures. The inability to understand hedge funds' investment strategies and to correctly attribute hedge fund returns to systematic risk factors is one of the main sources of ambiguity for their investors. Hence, hedge fund investors can easily be misguided when it comes to identifying "pure" hedge fund alphas. The second source of ambiguity in hedge fund returns relates to managerial skills. An investment in hedge funds is often considered a pure bet on the specific skills of a hedge fund manager; these skills are to a large extent characterized by probabilistic uncertainty. Finally, there is evidence of significant ambiguity affecting the dynamics of equity markets, as empirically documented by Anderson et al. (2009). One may therefore conjecture that systematic stock market ambiguity — or macroeconomic uncertainty — may affect equity hedge funds' expected returns. This conjecture is the

[2]See Epstein and Wang (1994) for a thorough discussion of this concept and its first application to asset pricing.

focus of our paper.

Whether hedge funds generate positive and significant alphas is widely debated in the academic literature and among practitioners. The disagreement is caused in part by the absence of an accepted asset pricing model and well-established systematic risk factors with respect to which performance and risks of hedge funds can be properly measured. In this study, we consider a new systematic factor which should be particularly important for measuring equity hedge funds' performance, namely systematic ambiguity. We postulate that beyond the traditional risk factors, equity hedge funds' expected returns embed a premium for "systematic ambiguity" exposure that should be priced in equilibrium. If systematic ambiguity exposure is ignored, alpha estimates may be biased and the performance of these hedge funds may be misread.

The paper consists of two parts: a theoretical presentation and an empirical study. In the theoretical part, we propose an asset allocation model for an ambiguity-averse equity hedge fund investor who makes her portfolio allocation decisions without relying on a single probability measure but rather by considering all feasible alternatives. The investor allocates her wealth among a risk-free bond, a risky stock (or broad stock market index), and an equity hedge fund by solving an intertemporal portfolio choice model in continuous time. Following Maenhout (2004), we explicitly incorporate ambiguity aversion into the utility function and assess the impact of ambiguity aversion on the optimal allocation solution. In the general model, we assume that ambiguity exists with respect to both hedge fund and stock market price dynamics. The ambiguity parameter corresponding to the stock market index price dynamics describes systematic ambiguity. The ambiguity parameter corresponding to the hedge fund price dynamics describes the investor's confidence regarding the hedge fund manager's skills.

We solve the asset allocation problem within a Max-Min utility framework to derive the optimal portfolio weights and consumption. We observe that, in general, ambiguity has a negative impact on the investor's allocation to risky assets. We next impose a market-clearing condition and derive a two-factor asset pricing model for the hedge fund in which the market risk premium arises because of standard risk aversion and a systematic market ambiguity premium arises because of ambiguity aversion. We call this model a capital asset pricing model with ambiguity (ACAPM). The main result from the theoretical section suggests that stock market ambiguity is priced in equilibrium and that this ambiguity premium may be hidden in equity hedge funds' alphas.

In the empirical part of the paper, we use the Trading Advisor Selection System (TASS) hedge fund database and focus exclusively on long/short equity hedge funds, since these funds have exposure only to stock market risk (and thus potentially to systematic ambiguity). We then raise two questions: First, do long/short equity hedge funds exhibit systematic ambiguity exposures? Second, can consideration of systematic ambiguity in the portfolio construction process enhance the out-of-sample performance of long/short equity hedge fund portfolios?

We measure systematic ambiguity by the dispersion (cross-sectional standard deviation) of the forecasts for the S&P 500 stock market index (for stock market ambiguity) and for the US Industrial Production Index (for macroeconomic ambiguity). We rely in both cases on a panel of survey-based forecasts from the Federal Reserve Bank of Philadelphia's Livingston Survey. In order to construct both systematic ambiguity factors, we estimate the stock market and macroeconomic ambiguity sensitivities of S&P 500 Index constituent stocks. We define the ambiguity factor as the spread between the out-of-sample returns of the portfolios consisting of the top decile and the bottom decile of stocks, respectively, as ranked by their stock market or macroeconomic ambiguity sensitivities.

To answer the first question, we compute the ambiguity betas for long/short equity hedge funds, which are estimated by adding the systematic ambiguity factor to various benchmark multifactor models: namely, to three well-known equity-based models (the CAPM and the Fama-French and Carhart models) and for robustness to a hedge fund-specific pricing model, specifically the Fung and Hsieh (2004) model with trend-following factors. The analysis is conducted for individual hedge funds and for equally weighted and value-weighted portfolios of hedge funds. We report significant estimates of ambiguity betas across most model specifications. As predicted in the theoretical section, we observe that ambiguity betas matter especially for those long/short equity hedge funds that pursue directional strategies.

To answer the second question, we analyze the out-of-sample performance of portfolios constructed with and without systematic ambiguity. We rank hedge funds based on their alphas (the hedge funds with top-decile alphas are included in the portfolio) from the CAPM with systematic ambiguity and compare these portfolios' out-of-sample performance with that of portfolios constructed using the traditional CAPM (without systematic ambiguity). We find that hedge fund portfolios constructed using the ACAPM outperform on an out-of-sample basis. The latter finding is particularly noticeable for value-weighted portfolios and holds for different hedge fund performance measurement models and holding periods.

The paper is organized as follows. Section 2.2 discusses the related literature. Section 2.3 describes the asset allocation model for ambiguity-averse hedge fund investors and the resulting two-factor ACAPM model with systematic ambiguity. Section 2.4 describes the data, the empirical methodology, and our results. Section 2.5 concludes the paper. Mathematical derivations, tables, and figures are provided as appendices in Section 2.6.

2.2 Related Literature

Our paper aims at contributing to the literature on ambiguity-averse preferences and Knightian uncertainty with a specific focus on hedge fund investments.

The formal incorporation of ambiguity into economic modeling requires constructing ambiguity-averse preferences. This development was pioneered by Gilboa (1987) and Gilboa and Schmeidler (1989), who built the axiomatic foundations of multiple prior preferences. Economic agents solve a Max-Min optimization problem by first minimizing their utility with respect to probability distributions from a given convex set (where the set of probabilities constitutes a menu of multiple priors corresponding to heterogeneous beliefs regarding the state of the economy) and then maximizing the expected utility of wealth with respect to the traditional set of variables (such as their consumption and investment choices). Another representation of ambiguity-averse preferences is developed in papers by Hansen and Sargent (2001) and Anderson et al. (2003), who describe utility optimization as a robust control problem. This problem also has a Max-Min optimization form with a minimization over alternative probability measures, but the utility function contains a penalty in terms of the entropy measure relative to alternative probability laws. It has been shown[3] that the robust control problem is equivalent to the multiple priors setting only in the case of constrained relative entropy. This robust control approach offers a very appealing and intuitive interpretation of the penalization term; however, it may fail to satisfy some axiomatic foundations of economic preferences for general specifications of the penalty function. Maccheroni et al. (2006) have resolved this potential inconsistency by constructing an ambiguity-averse utility function of a general class that encompasses both the multiple priors and the robust control settings, and therefore makes the latter model consistent with axioms of economic preferences.

A body of literature has examined whether ambiguity-averse preferences can explain some well-known financial market anomalies. Dow and Werlang (1992) utilize multiple prior preferences to conclude that ambiguity aversion can explain limited participation in equity markets. A recent paper by Easley and O'Hara (2009) also demonstrates that ambiguity aversion induces nonparticipation in financial markets and suggests that regulation that tends to decrease perceived ambiguity, especially during disruptive market events, can help mitigate the effect of ambiguity aversion and thus resolve the non-participation puzzle. The authors use as an example the 2008 credit crunch crisis, when governments

[3]See for example Trojani and Vanini (2004).

all over the world increased the sums of insured deposits and indicated their willingness to bail out major financial corporations at the verge of bankruptcy in order to diminish ambiguity-induced lack of market participation.

Epstein and Miao (2003) address the equity home-bias puzzle (underinvestment in foreign securities) and the consumption home-bias puzzle (high correlation between country-specific consumption growth and output growth) by using a multiple-prior utility function to define ambiguity-averse preferences. The authors propose a heterogeneous agent model of an exchange economy in complete markets. The agents differ not only in their endowments but also in their ambiguity aversions (with higher ambiguity towards foreign markets). The paper presents a closed-form equilibrium solution that shows that an endogenously determined ambiguity or disagreement process appears in the equilibrium prices and allocations. The equity home bias is explained by the difference in beliefs generated by the disagreement term. In contrast to a standard heterogeneous-agent model with heterogeneity in beliefs or risk preferences, the main contribution of the Epstein and Miao (2003) model is that the disagreement term is derived endogenously due to agents' heterogeneous ambiguity aversions. Uppal and Wang (2003) also explain the home bias effect in agents' asset allocations by using a robust control problem to account for ambiguity. The authors argue that the robust control approach not only allows multiple priors but also establishes a reference model with respect to which an agent can differentiate among the priors instead of always picking the worst-case scenario. The optimal portfolio allocation is presented as an extension of the standard Merton portfolio weights. The paper conducts a model calibration and shows that when the overall degree of ambiguity is high, the home bias in the optimal holdings is stronger.

The equity premium puzzle can also be meaningfully addressed by ambiguity-averse preferences, since ambiguity aversion raises overall risk aversion and therefore the equity premium. This result is derived by Maenhout (2004), who obtains a closed-form solution for the portfolio choice problem in continuous time for i.i.d. returns. The author proposes using a state-dependent weighting function in the penalty term of the robust control problem to solve the optimization problem analytically and to preserve wealth independence in the optimal solution. Maenhout (2004) emphasizes the decrease (up to 50% for reasonable calibration parameters) in demand for the risky asset by robust investors that leads to an increase in the equity premium and to a drop in the risk-free interest rate. The equity premium is calibrated to lie between 4% and 6% when the model considers both risk aversion and ambiguity aversion. We will rely on a similar formulation of the portfolio choice problem and use a similar functional form for the penalty term in our hedge fund asset allocation model.

As a starting point for our empirical analysis, we follow the results stemming from the theoretical model proposed by Kogan and Wang (2003), a two-factor asset pricing model based on a return versus risk and ambiguity relationship. To the best of our knowledge, the only previous empirical test of an ambiguity-based asset pricing model was undertaken by Anderson et al. (2009), inspired by the asset pricing model derived by Kogan and Wang (2003). The former authors construct a measure of macroeconomic ambiguity as the dispersion in the forecasts of nominal GDP growth and of corporate profits after taxes. The authors find that macroeconomic ambiguity is indeed an additional priced factor that has a significant impact on stocks' expected returns.

Krahnen et al. (2012) estimate ambiguity aversion by conducting experiments under various settings and observing how reservation prices of individuals vary with ambiguity. The authors conclude that ambiguity aversion exists and differs across individuals. They demonstrate that the ambiguity effect can be separated from the effect of risk aversion and that ambiguity aversion has a stronger impact on asset prices than risk aversion. Therefore, accounting for ambiguity is important in many financial applications, especially in asset pricing.

Within the hedge fund literature, the identification of risk-adjusted performance is a widely discussed but still challenging topic due to the absence of a proper hedge fund pricing model and due to the dynamic and nontractable risk exposures of these funds. One can compare the ambiguity approach with Bayesian methods[4] to estimate alphas as shown by Kosowski et al. (2007). They apply a nonparametric bootstrap analysis to estimate alphas of hedge funds, relying on the fact that most estimates of alpha fail to fit the normal distribution and exhibit significant negative skewness and high kurtosis. They take the seven-factor model of Fung and Hsieh (2004) as a benchmark risk model. The bootstrap results indicate that ordinary least squares (OLS) alphas are often overstated and have low persistence. The Bayesian alpha is found to be positive and statistically significant at annual horizons; thus, according to the authors, hedge funds' performance cannot be attributed to luck. Avramov et al. (2011) further explore the Bayesian approach and study the performance of hedge funds while assuming predictability in their return-generating model. The authors find that strategies incorporating predictability in managerial skills outperform substantially. Hence, the authors claim that predictability in alphas explains a large component of hedge fund returns as well as the cross-sectional dispersion observed in their performance.

Cvitanic et al. (2003) derive a closed-form solution to the optimal hedge fund allocation problem for investors with constant relative risk aversion (CRRA) utility and in the presence of uncertain abnormal returns, for instance with Gaussian priors on the abnormal returns (relative to the CAPM alphas) of risky assets. The authors estimate uncertainty risk as the standard deviation of alpha estimates across different asset pricing models. They find that the optimal portfolio weights allocated to hedge funds should be lower under model misspecification than under the standard optimal asset allocation framework.

Finally, the recent paper by Detemple et al. (2010) proposes an asset allocation model in which hedge funds represent a nonredundant asset class. They analyze both theoretically and empirically the market price of hedge fund risk, and find that including hedge funds

[4] Note that a Bayesian method implies a single prior framework while ambiguity involves a multiple priors approach.

in the optimal asset allocation strategy can provide substantial economic benefits due to the presence of hedge fund managers with market-timing abilities.

2.3 An Equity Hedge Fund Asset Allocation Model under Ambiguity

This section describes the portfolio allocation model of an equity hedge fund investor with ambiguity-averse preferences. The investment opportunity set consists of three assets: a risk-free asset and two risky assets, a stock representing the market portfolio and a hedge fund that can itself take long or short positions in the stock market. The investor has CRRA preferences and simultaneously displays ambiguity aversion. We investigate the impact of ambiguity aversion on the optimal portfolio allocation. Furthermore, we derive the equilibrium pricing implications of the model. In particular, we find that only systematic ambiguity over the market portfolio returns is priced in equilibrium. The resulting equilibrium pricing model, the ACAPM, is a two-factor model in which systematic risk and ambiguity are both priced.

2.3.1 Assets

Three types of assets exist in the economy. The first asset is a risk-free bond of price B_t, paying a constant instantaneous interest rate r, implying the dynamics:

$$dB_t = rB_t dt. \tag{2.1}$$

The second asset is a risky stock market portfolio whose instantaneous returns $\frac{dM_t}{M_t}$ follow a geometric Brownian motion with a constant drift μ_M and a constant volatility σ_M:

$$\frac{dM_t}{M_t} = \mu_M dt + \sigma_M dZ_t^M, \tag{2.2}$$

where dZ_t^M is the increment of a standard Brownian motion.

The third asset is an equity hedge fund whose instantaneous returns follow a geometric Brownian motion with a constant drift μ_F and a constant volatility σ_F:

$$\frac{dF_t}{F_t} = \mu_F dt + \sigma_F d\widehat{Z}_t. \tag{2.3}$$

The hedge fund can invest in both the risky and the risk-free assets, taking either long or short positions. The total risk of investing in the hedge fund can be broken down into the systematic risk arising from its exposure to the stock market and into idiosyncratic hedge fund risk. The standard Brownian motion $\widehat{Z}_t$ represents the total risk of the hedge fund and satisfies the following equation:

$$d\widehat{Z}_t = \rho dZ_t^M + \sqrt{1-\rho^2} dZ_t^F, \tag{2.4}$$

where ρ is the correlation coefficient between the hedge fund and stock market returns, Z_t^F is a Brownian motion related to the idiosyncratic (hedge fund-specific) risk, and Z_t^M is the Brownian motion mentioned above, relating to the systematic risk (market portfolio).

We assume that the correlation coefficient between the hedge fund and the stock market returns is constant:

$$\mathbb{E}(dZ_t^F dZ_t^M) = \rho dt. \tag{2.5}$$

Different hedge fund investment strategies can be distinguished from each other by the values of their correlation coefficients. A higher correlation is attributed to directional strategies while a lower value of the coefficient would characterize “market-neutral” strategies.

Constant model parameters — such as volatilities or correlations — may represent a strong assumption, given the time-varying risk exposures typically taken by hedge funds. Even though stochastic models with the above-mentioned state variables might better describe the dynamics of hedge fund returns under partially observable, dynamic risk exposures, we try to keep the modeling framework parsimonious to focus on the ambiguity-related implications for the cross-section of equity hedge funds' returns.

2.3.2 Model Misspecification

We assume that the model or probability law that characterizes the stochastic dynamics of risky assets returns is not correctly specified. Let $\mathcal{P}$ be the initial probability measure under which the assets returns' dynamics in the economy are specified. We refer to the stochastic equations describing the dynamics of asset returns under this probability measure as the reference model. We denote the alternative probability measures by $\mathcal{Q}^H$, parameterized by an appropriately adapted process H_t. The existence of the process H_t is ensured by Girsanov's Theorem. The process H_t uniquely defines the alternative probability measures. We assume that $\mathcal{Q}^H$ is an absolutely continuous measure with respect to the reference probability measure $\mathcal{P}$, so that the Radon-Nikodym derivative or density $\frac{d\mathcal{Q}^H}{d\mathcal{P}}$ exists and is correctly defined.

Moreover, the density coincides with its conditional expectation. Under Novikov's condition, the density is an exponential martingale:

$$\frac{d\mathcal{Q}^H}{d\mathcal{P}} = \exp\left\{-\int_0^T \frac{|H_t|^2}{2}dt - \int_0^T H_t dZ_t\right\}, \tag{2.6}$$

where $Z_t = (Z_t^M, Z_t^F)$ is a vector of Brownian motions and $H_t = (h_t^M, h_t^F)$ is a vector of ambiguity parameters related to the corresponding sources of ambiguity. Since H_t is a vector, we have $|H_t|^2 = (h_t^M)^2 + (h_t^F)^2$. Indeed, we assume that there is no correlation between the two sources of ambiguity h_t^M and h_t^F. This equation determines the parameterization of alternative probabilities. Since we have two sources of risk, the stock market risk and the idiosyncratic hedge fund risk, we also obtain two sources of ambiguity: the systematic stock market ambiguity and the idiosyncratic hedge fund ambiguity.

H_t determines the relationship between the Brownian motions related to the reference and alternative models:

$$Z_t^H = Z_t + \int_0^t H_s ds, \tag{2.7}$$

where Z_t^H is a vector of Brownian motions related to the alternative model and Z_t is the vector of Brownian motions defined above, relating to the reference model. Given the parametrization of the alternative probability measure, the following lemma holds:

Lemma 2.1. *The dynamics of stock market returns and of the long/short equity hedge fund returns under the alternative probability measure $\mathcal{Q}^H$ jointly satisfy*

$$\frac{dM_t}{M_t} = \left(\mu_M + \sigma_M h_t^M\right) dt + \sigma_M dZ_t^M, \tag{2.8}$$

$$\frac{dF_t}{F_t} = \left(\mu_F + \rho\sigma_M h_t^M + \sigma_F \sqrt{1-\rho^2} h_t^F\right) dt + \sigma_F \left(\rho dZ_t^M + \sqrt{1-\rho^2} dZ_t^F\right). \tag{2.9}$$

The stochastic dynamics of assets' returns under the alternative probability measure are determined only by the drift change of each corresponding process scaled by the volatility coefficients. The details of the proof can be found in Appendix 2.6.1.

We use robust control optimization with a relative entropy penalty term to specify ambiguity aversion. The relative entropy measures the size of model misspecification or the "distance"[5] between two probability laws $\mathcal{Q}^H$ and $\mathcal{P}$, and is defined as follows:

$$Ent(\mathcal{Q}^H|\mathcal{P}) = \mathbb{E}^H \ln\left(\frac{\mathcal{Q}^H}{\mathcal{P}}\right). \tag{2.10}$$

Lemma 2.2. *The local relative entropy has the following functional form:*

$$Ent(\mathcal{Q}^H|\mathcal{P}) = \mathbb{E}^H \frac{1}{2}\int_0^T H_t^2 dt. \tag{2.11}$$

[5]Strictly speaking, the entropy is not a true distance measure because it is not symmetric.

The global entropy is a compounded local relative entropy:

$$Ent^{glob}(\mathcal{Q}^H|\mathcal{P}) = \delta \int_0^\infty e^{-\delta t} \left(\mathbb{E}^H \int_0^t \frac{H_s^2}{2} ds \right) dt, \tag{2.12}$$

where $\delta > 0$ is a discount factor.

Lemma 2.2 (details of the proof can be found in Appendix 2.6.1) shows that entropy is a convex function of the ambiguity parameters captured in H_t at any time t.

2.3.3 Investor Preferences and the Portfolio Optimization Problem

The investor is characterized by a CRRA intertemporal utility over an infinite time horizon, with a discount factor δ and a risk-aversion coefficient γ. The entropy in the intertemporal utility of an ambiguity-averse agent constitutes a penalty term for any deviation from the reference model. The utility optimization problem has a Max-Min form, according to which we first minimize with respect to parameters of the alternative probability laws h^M and h^F, searching for the worst-case scenario, and then maximize expected discounted utility with respect to consumption and the portfolio weights:

$$\max_{\theta_M,\theta_F,C} \min_{h^M,h^F} \mathbb{E}^H \int_0^\infty e^{-\delta t} \left(\frac{C_t^{1-\gamma}}{1-\gamma} + \frac{(h_t^M)^2}{2\psi^M} + \frac{(h_t^F)^2}{2\psi^F} \right) dt, \tag{2.13}$$

where C_t is the investor's instantaneous consumption, θ_M is the fraction of his wealth invested in the stock market, θ_F is the fraction of his wealth invested in the hedge fund, and the residual $1 - \theta_M - \theta_F$ is allocated to the risk-free bond. (h_t^M, h_t^F) is a vector of ambiguity parameters related to stock market ambiguity and hedge fund ambiguity. The positive vector parameter $\psi = (\psi^M, \psi^F)$ is an agent-specific weight indicating how much the investor penalizes the alternative scenarios. This parameter represents the ambiguity-aversion coefficient. If $\psi = 0$, the deviation penalty is infinite and the investor chooses to remain under the reference model. If $\psi \to \infty$, the penalty term goes to zero in the limit, and the investor does not restrict himself in the choice of alternative probability measures. This is a myopic solution.

The optimization problem with the penalty function is solved subject to the stochastic wealth dynamics of the investor endowed with initial wealth W_0 as in the standard Merton model.

Theorem 2.1. *The solution of the optimization problem (2.13) subject to stochastic dynamics of asset returns as described in Lemma 2.1 for an ambiguity-averse agent with initial wealth W_0 is the following:*

- *the optimal consumption is*

$$C^* = [J_W]^{-\frac{1}{\gamma}}, \tag{2.14}$$

  where $J(W,t)$ is the indirect utility function satisfying the Hamilton-Jacobi-Bellman (HJB) equation;

- *the optimal fraction of wealth invested in the stock market is*

$$\theta_M = \frac{\mu^M - r - \theta_F \rho \sigma_M (\Omega_M \sigma_M + \gamma \sigma_F)}{(\gamma + \Omega_M)\sigma_M^2}; \tag{2.15}$$

- *the optimal fraction of wealth invested in the hedge fund is*

$$\theta_F = \frac{\mu^F - r - \theta_M \rho \sigma_M (\Omega_M \sigma_M + \gamma \sigma_F)}{\gamma \sigma_F^2 + \Omega_M \rho^2 \sigma_M^2 + \Omega_F \sigma_F^2 (1 - \rho^2)}, \tag{2.16}$$

  where $\Omega = (\Omega^M, \Omega^F)$ is a time-invariant vector proportional to the ambiguity aversion vector-coefficient $\psi = (\psi^M, \psi^F)$:

$$\psi = \frac{\Omega}{W J_W}. \tag{2.17}$$

Proposition 2.1. *The explicit solution for the optimal portfolio weights is as follows:*

$$\theta_M = \frac{(\mu^M - r) - \frac{B}{D}(\mu^F - r)}{A - \frac{B^2}{D}}, \tag{2.18}$$

$$\theta_F = \frac{(\mu^F - r) - \frac{C}{A}(\mu^M - r)}{D - \frac{B^2}{A}}, \tag{2.19}$$

where coefficients A, B, C, and D are parameters dependent on volatilities σ_M and σ_F, ambiguities Ω_M and Ω_F, and the correlation coefficient ρ:

$$A = (\gamma + \Omega_M)\sigma_M^2, \tag{2.20}$$

$$B = \rho\sigma_M(\Omega_M\sigma_M + \gamma\sigma_F), \tag{2.21}$$

$$C = B, \tag{2.22}$$

$$D = \gamma\sigma_F^2 + \Omega_M\rho^2\sigma_M^2 + \Omega_F\sigma_F^2(1 - \rho^2). \tag{2.23}$$

The derivation of the optimal portfolio weights can be found in Appendix 2.6.1.

The influence of systematic stock market ambiguity and idiosyncratic hedge fund ambiguity on the optimal portfolio choice depends on the correlation coefficient ρ. On the one hand, when ρ approaches 1, $\Omega_F = 0$ and the impact of hedge fund ambiguity disappears. The stock market ambiguity affects the optimal portfolio weights either by increasing overall risk aversion (denominator effect) or through relative betas $\beta_{M,F} = \frac{B}{D}$ and $\beta_{F,M} = \frac{C}{A}$. On the other hand, when ρ approaches 0, the relative betas are equal to zero (there is no correlation between hedge fund and stock market returns: $\beta_{M,F} = \beta_{F,M} = 0$) and the formulas for the optimal allocation reduce to the following expressions:

$$\theta_M = \frac{\mu^M - r}{(\gamma + \Omega_M)\sigma_M^2}, \tag{2.24}$$

$$\theta_F = \frac{\mu^F - r}{(\gamma + \Omega_F)\sigma_F^2}. \tag{2.25}$$

Equations (2.24) and (2.25) can be compared with Merton's optimal portfolio weights in the case of ambiguity-averse preferences. In this case, ambiguity simply amplifies the impact of risk aversion, reducing the optimal portfolio allocation to risky securities.

2.3.4 Equilibrium Equity Hedge Fund Pricing Model

To derive the equilibrium pricing relationship, we now impose the market-clearing conditions on the optimal portfolio weights. In equilibrium, the representative investor holds the stock market portfolio. The market-clearing conditions for the portfolio weights thus satisfy the following relation:

$$\theta_F = 0 \text{ and } \theta_M = 1. \tag{2.26}$$

Substituting those values into the implicit expressions for the optimal portfolio weights in Theorem 2.1, we obtain the following proposition:

Proposition 2.2. *The expected stock market returns satisfy the following condition:*

$$\mu^M = r + \gamma\sigma_M^2 + \Omega_M\sigma_M^2. \tag{2.27}$$

The expected hedge fund returns satisfy the following:

$$\mu^F = r + \gamma\sigma_F\rho\sigma_M + \Omega_M\rho\sigma_M^2. \tag{2.28}$$

Proposition 2.2 defines a representation of the ACAPM for expected stock market and hedge fund returns. These equations show that the investor is compensated both for bearing stock market risk and for acting under stock market ambiguity. These formulas are similar to the two-factor relationship between equilibrium expected returns and risk and ambiguity that were first derived by Kogan and Wang (2003) and empirically tested by Anderson et al. (2009) for the equity market. Our paper empirically relies on this proposition to characterize long/short equity hedge fund returns.

The risk and ambiguity premia depend on the correlation coefficient between hedge fund and stock market returns. Under the assumption of zero correlation, hedge fund investors would not earn a premium for either stock market risk or stock market ambiguity. Such a relationship is characteristic of nondirectional hedge fund strategies and absolute return strategies with zero market betas. If the correlation is nonzero, the investor in the equity hedge fund also earns an ambiguity premium reflected in the term $\Omega_M \rho \sigma_M^2$ as well as a stock market risk premium expressed by the well-known term $\gamma \sigma_F \rho \sigma_M$. In the empirical section, we organize the sample of long/short equity hedge funds based on their equity market betas, assign them to the categories of directional and nondirectional equity hedge funds, and separately estimate their ambiguity exposures. We conjecture, based on Proposition 2.2, that directional hedge funds should have higher stock market ambiguity exposures.

2.4 Empirical Analysis

The objective of the empirical analysis is twofold. First, we estimate the ambiguity betas of long/short equity hedge funds with various benchmark models to compare their systematic ambiguity exposures with our theoretical model's predictions. Second, we construct long-only portfolios of long/short equity hedge funds relying on the ACAPM alphas in the portfolios' formation, and compute these portfolios' out-of-sample returns. We then compare the performance of these portfolios with that of portfolios formed without taking systematic ambiguity exposure into account.

This empirical study is, to our knowledge, the first attempt to estimate the impact of systematic ambiguity on long/short equity hedge funds' return properties. We propose to construct systematic ambiguity factors based on the cross-sectional dispersion of professional forecasts for stock market index returns and a specific macroeconomic indicator, namely industrial production growth.

The analysis is restricted to long/short equity hedge funds for two reasons. First, this is the largest category of hedge funds — in terms of assets under management — within our hedge fund sample. Second, the primary investment instruments used in this strategy are equity-linked and thus allow us to focus meaningfully on the role of stock market ambiguity on the returns of hedge funds belonging to this strategy. We do not exclude the possibility that ambiguity also affects other hedge fund strategies. However, the return-generating model would then require different benchmark pricing models and more detailed knowledge of the specific investment instruments used by those hedge funds. This issue is left for future research.

2.4.1 Hedge Fund Data

We use the TASS Hedge Fund Database during the period from January 1994 through November 2007.[6] The database contains monthly data on the rate of return, assets under management, and other characteristics of live and defunct hedge funds. A defunct hedge fund is a fund that stopped reporting to the TASS database due to liquidation, merger, or any other reason. These funds are contained in the Graveyard module, which is available from January 1994 onwards.[7] Reporting to the database is a voluntary decision for a hedge fund's manager, and thus we cannot know the exact reason for nonreporting. Nevertheless, the combination of defunct and live funds allows us to account for survivorship bias.

We impose various filters to refine our data sample. First, we exclude the first 12 months of observations from the sample in order to account for instant history bias. Backfill or instant history bias is common in hedge fund databases as fund managers choose to report hedge fund performance, not from inception, but rather after an established successful track

[6]The data for the paper were purchased at the end of 2007. We do not update the paper from 2008 onwards. We believe that the main conjectures are still valid after 2008, especially as investors have demonstrated significant ambiguity aversion since the crisis. However, investors exited both the equity market and equity hedge funds in the fall of 2008. Therefore, the implications of the asset allocation model in the paper should be tested on the complete set of asset classes and hedge fund strategies. This task is left for future research.

[7]This is the reason why most research using this database starts in January 1994.

record, backfilling missing historical data. Second, only hedge funds reporting returns net of fees in U.S. dollars and at monthly tracking frequency are selected. The final sample of long/short equity hedge funds over the period between January 1994 and November 2007 consists of 2070 observations. The average (mean) life time of a long/short equity hedge fund in the sample is 60 months, while the median lifetime is 50 months.

Panel A of Table 2.1 (see Appendix 2.6.2 for all tables) presents descriptive characteristics of our sample of hedge funds. We report cross-sectional average values computed from the monthly data for the whole sample and for each year.[8] The number of long/short equity hedge funds grew from 206 in 1994 to 986 in 2007, with a peak of 1168 hedge funds in 2005. The positive trend in the sample size reflects the rapid development of this segment of the hedge fund industry over the 13 years covered by our data. The drop in the sample size during the last year may indicate the realization of capacity constraint in the long/short equity hedge fund strategy. Moreover, many quantitative equity funds experienced large losses during August 2007. The average assets under management (AUM) of hedge funds grew from USD 60M to USD 152M by the end of 2007, albeit with troughs in 1995 and 1999.

The fee structure consists of a fixed management fee, an incentive fee, and a high water mark provision. The management fee is on average 1.20%; the average incentive fee reaches 18% during the sample period. In parallel, the lockup period[9] increased from 2.8 to 5.5 months during the sample period. The leverage ratio reached its peak of 140% in 1999 and 2000 and then started decreasing to about 130%. However, the number of hedge funds reporting nonzero leverage values increased considerably during our sample period. Roughly one-third of long/short equity hedge funds reported nonzero leverage over the sample period.

Panel B of Table 2.1 reports descriptive statistics of long/short equity hedge fund

[8] Note that we only cover the first 11 months in 2007.

[9] The lockup period is a certain period of time during which the money allocated to the hedge fund cannot be withdrawn.

returns. Long/short equity hedge funds display large variability in their monthly returns: the annualized average hedge fund return is 12.9% with an annualized standard deviation of 15.92%. The annualized average Sharpe ratio[10] is 0.70. The annual return distribution is positively skewed (the skewness coefficient is 0.76). However, the annual kurtosis coefficient is close to the value of 3 for the normal distribution: the mean and median values are 4.25 and 3.56 respectively, with a standard deviation of 1.58. In addition, we performed a Jarque-Bera goodness-of-fit test of the normality of hedge fund returns. The null hypothesis indicates that the returns are normally distributed against the alternative hypothesis that the returns are not normally distributed (within the Pearson family of distributions). We run the test for each hedge fund and assign a value $JB = 0$ if the null hypothesis cannot be rejected and $JB = 1$ otherwise. We report the cross-sectional average of JB values as well the median value and the standard deviation. $JB = 0.66$ means that we reject the null hypothesis of a normal distribution of returns for 66% of the long/short equity hedge funds in the sample. Moreover, we report the cross-sectional average, median, and standard deviation for the p-values and actual values of the Jarque-Bera statistic.

2.4.2 Construction of the Systematic Ambiguity Factors

We construct two systematic ambiguity factors using the dispersion (standard deviation) in the cross-section of survey-based forecasts for S&P 500 stock market index returns and for growth in the US Industrial Production Index. The data source for the forecasts is the Livingston Survey, begun in June 1946 by Joseph Livingston and taken over in 1990 by the Federal Reserve Bank of Philadelphia. The survey, conducted semiannually each June and December, asks participants to provide forecasts for key economic variables for the end of the current month, 6 months ahead, and 12 months ahead. We will consider

[10] The Sharpe ratio is defined as the ratio of the historic average hedge fund monthly return in excess of the risk-free rate (the return on a 1-month Treasury bill) to the standard deviation of monthly returns. The Sharpe ratio is annualized and a cross-sectional mean value is reported.

only the medium term 6-months-ahead forecasts for the analysis. On the one hand, the forecasts for the end of the current month might be inappropriate as some information about the actual values of the variables in the current month may be learned during the month by a forecaster. On the other hand, the cross-section of the longer-term 12-months-ahead forecasts might have higher dispersion, not because of ambiguity regarding the state of economy but rather due to a greater inability to forecast long-term trends. The affiliations of individual forecasters vary and consist of nonfinancial business (30%), investment banking (29%), commercial banking (20%), academic institutions (13%), and government, insurance companies, and labor organization (the remaining 8%).[11]

We will rely on two different systematic ambiguity measures: macroeconomic ambiguity and stock market ambiguity. We use these two measures for two reasons. On the one hand, using two different variables would contribute to the robustness of the results and mitigate potential data problems with survey-based forecasts. On the other hand, we believe that both macroeconomic and stock market ambiguity may significantly affect the returns of long/short equity hedge funds, and we are interested in the comparative analysis for those two ambiguity measures.

As far as the macroeconomic ambiguity measure is concerned, the Livingston Survey provides values for the 6-months-ahead forecast for the Industrial Production Index with seasonal adjustments. The data are taken from December 1989. Actual values of the Industrial Production Index are available from the Federal Reserve Board G17 statistical release series. On average, the panel of respondents consists of 37 forecasters, with a standard deviation of 12. We compute the expected semiannual index return as the percentage change between a forecast and a base-period index level. The macroeconomic ambiguity measure is defined as the cross-sectional standard deviation of the forecasted returns.

The stock market ambiguity measure is defined as the standard deviation in the forecasts of S&P 500 stock market index returns. The stock market forecast is a 6-months-

[11]See Croushore (1997) for further details on these estimates.

ahead forecast for the S&P 500 Index level on the last trading day of June or December. The underlying data source is the *New York Times*, and the data are available from December 1990. On average, the panel of respondents consists of 25 forecasters, with a standard deviation of 6. We compute a semiannual expected return for each forecaster as the percentage price return between a forecasted index value and a base-period index level. The ambiguity measure is again defined as the cross-sectional standard deviation of the forecasted returns.

To construct each ambiguity factor, we proceed as follows: we rank all constituents of the S&P 500 Index[12] according to their sensitivity to the specific ambiguity measure (S&P 500 or Industrial Production) by running the following OLS rolling regression at semiannual frequency and with a fixed-size estimation window:

$$StockReturn_{i,t} - RiskFree_t = \alpha + \beta_{mkt}(SP500_t - RiskFree_t) + \beta_{amb}Amb_t + \varepsilon_t. \quad (2.29)$$

The ambiguity factor is then computed as the out-of-sample return of an equally weighted long/short portfolio of stocks where long positions are taken in the top decile and short positions are taken in the bottom decile of stocks ranked by their ambiguity sensitivities β_{amb}. The size of the estimation window is 4 years. The initial estimation window consists of all data prior to January 1994: from December 1990 for stock market ambiguity and from December 1989 for macroeconomic ambiguity. In both cases, the first observation for the ambiguity factor refers to January 1994. We generate monthly ambiguity factor returns while we re-estimate the regression for the sensitivities every 6 months.

The time series of ambiguity factor returns for the stock market and macroeconomic ambiguity factors are displayed in Figure 2-1 in Appendix 2.6.2. In both cases, the ambiguity factor experiences the highest volatility over the period between 2000 and 2003. The maximum absolute values of the ambiguity factors are reached in 2000, during the spikes

[12] Data for stock returns of S&P 500 constituent stocks are downloaded from the CRSP (Center for Research in Security Prices) database.

in uncertainty just before the dot-com bubble burst. The lowest values of the ambiguity factors are usually observed during periods of economic recovery.

In order to investigate potential multicollinearity among the two ambiguity factors and the other standard risk factors, we calculate pairwise correlation coefficients: the Pearson correlation, the Kendall correlation, and the Spearman correlation for both the macroeconomic ambiguity *AmbIP* and the stock market ambiguity *AmbSP* factors and the Fama-French and Carhart momentum risk factors. The correlation coefficients are displayed in Panel A of Table 2.2. We observe that both ambiguity factors have low correlation levels with the equity market factor. It is worth noting, however, that we found different signs for the correlations of the market factor with the stock market ambiguity factor (positive) and with the macroeconomic ambiguity factor (negative), respectively. Higher correlations are observed with the Fama-French factors *HML* (high minus low) and *SMB* (small minus big). *SMB* and *HML* have relatively high Pearson correlations with the stock market ambiguity factor: 0.37 and -0.37, respectively. However, for the Kendall and Spearman coefficients, the values drop to negligible levels. Hence, it is likely that the high value of the Pearson correlation is due to some outliers and does not present a matter of concern. The momentum factor, however, exhibits strong correlation with the macroeconomic ambiguity factor: 0.65 for the Pearson coefficient and 0.43 for the Spearman coefficient. The correlation between momentum and stock market ambiguity is low: 0.17 for the Pearson coefficient and 0.07 for the Spearman coefficient.

Given the high correlations observed between the macroeconomic ambiguity and momentum factors, we further investigate the causality between those factors. Panel B of Table 2.2 reports values of the F-statistic and corresponding critical values of the Granger causality test with a maximum of 5 lags (the actual number of lags is chosen using the Bayesian information criterion) and at the 5% significance level. The values F and Fc correspond to the regressions such that momentum causes ambiguity and the values G and Gc correspond to the opposite such that ambiguity causes momentum. Whenever the value of the F-statistic is higher than the critical value, we reject the null hypothesis of no Granger causality. We reject the null hypothesis in both directions in the case of the stock market ambiguity *AmbSP* factor. However, we cannot reject the hypothesis that momentum causes ambiguity in the case of the macroeconomic ambiguity factor *AmbIP*.

2.4.3 Long/Short Equity Hedge Funds' Ambiguity Betas

In this section, we estimate the ambiguity exposures of long/short equity hedge funds by adding the macroeconomic or stock market ambiguity factor to traditional linear multi-factor hedge fund return-generating models. The regressions are estimated for portfolios of equally weighted and value-weighted long/short equity hedge funds as well as for individual long/short equity hedge funds. All coefficients are estimated with standard OLS regressions.

We will rely on three benchmark risk factor models: the CAPM, the Fama-French model augmented with the *HML* and *SMB* factors, and the Carhart model augmented with the momentum factor. The data for the equity factors are taken from the K. French data library.[13] The market factor is the return on the market (the value-weighted return on all NYSE, AMEX, and NASDAQ stocks) in excess of the risk-free rate. The risk-free rate is the 1-month T-bill rate taken from Ibbotson Associates.

Let us start by examining the estimated ambiguity betas for the equally weighted (EW) and value-weighted (VW) long/short equity hedge fund portfolios. The estimates are presented in Table 2.3 for the CAPM and the Fama-French and Carhart models, augmented by the ambiguity factors. Under the ACAPM, we find significant estimates of ambiguity betas in the range of 0.07 to 0.15, depending on the portfolio formation and type of ambiguity factor. Value-weighted portfolios tend to have higher ambiguity betas, suggesting that

[13] I thank K. French for providing these risk factors. Data are publicly available and regularly updated on the following website: `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

larger hedge funds display higher stock market or macroeconomic ambiguity exposures. Ambiguity betas are slightly higher for macroeconomic ambiguity that for stock market ambiguity. The market betas for the portfolios of hedge funds are fairly stable with values around 0.50.

The Fama-French and Carhart models both have higher explanatory power for long/short equity hedge fund portfolio returns than the one-factor CAPM: 0.83 and 0.85, respectively, versus 0.68 for the equally weighted portfolios and 0.70 and 0.77, respectively, versus 0.50 for the value-weighted portfolios. The ambiguity betas in the Fama-French specification are lower than in the CAPM and are significant only for the macroeconomic ambiguity factor. We document significant estimates of ambiguity betas of 0.05 and 0.10 for the equally weighted and value-weighted portfolios, respectively. In the case of the Carhart 4-factor model, ambiguity betas lose their statistical significance while the momentum factor is highly significant. The likely cause of this insignificance is the potential multicollinearity between the macroeconomic ambiguity factor and the momentum factor reported in Table 2.2 and discussed in Section 2.4.2.

We then extend the measurement of ambiguity exposures to individual long/short equity hedge funds, running each of the three multifactor risk models with both ambiguity factors for each individual hedge fund in the sample. Table 2.4 reports the mean and median estimates of alphas and ambiguity betas that are significant at the 10% level as well as the adjusted R^2 coefficients for these regressions in the case of both macroeconomic ambiguity and stock market ambiguity factors. We observe that about 27% of long/short equity hedge funds have significant stock market ambiguity exposures and 32% have significant macroeconomic ambiguity betas under the ACAPM regression. The median estimate of the ambiguity beta equals 0.13 and 0.15 for the stock market and macroeconomic ambiguity factors, respectively. The median explanatory power of the models with ambiguity for individual hedge funds reaches approximately 0.30 in the case of the augmented CAPM. Ambiguity betas and the number of hedge funds with significant ambiguity betas relative to the augmented Fama-French and Carhart models are lower. However, in these cases, we did not adjust the results for the multicollinearity between the ambiguity factors and the momentum factor.

Next, we differentiate hedge funds by their market betas and examine separately the ambiguity betas for high and low market beta hedge funds. These results are reported in the bottom two panels of Table 2.4. The threshold for the market betas is set at 0.10. High market beta hedge funds are those with estimated values of betas higher than 0.10 and statistically significant at 10% level. Low market beta hedge funds constitute the rest of the sample. There are 838 hedge funds in the high market beta category and 1232 in the low market beta category. About 50 to 56% of the high market beta hedge funds (446 hedge funds for the ACAPM model) have significant ambiguity exposures with estimated ambiguity beta coefficients similar to those observed in the overall sample: 0.13 for both the stock market and macroeconomic ambiguities. Only 12 to 18% of the low market beta hedge funds exhibit significant exposure to the ambiguity factor with mean estimates of the ambiguity beta of 0.05 for the stock market ambiguity factor in the ACAPM model. These results confirm the hypothesis stated in the theoretical section, namely that systematic ambiguity matters primarily for long/short equity hedge funds that pursue directional strategies and thus have higher stock market betas.

We further test whether the macroeconomic ambiguity factor adds value relative to the Carhart model. For this purpose, we augment the Carhart model with macroeconomic ambiguity by substituting the original MOM (momentum) and ambiguity factors by their orthogonal counterparts: the principal components $PC1$ and $PC2$ of the joint matrix [MOM,$AmbIP$]. By considering principal components that are orthogonal, by definition, we solve the potential multicollinearity issue. Assuming that the orthogonal momentum factor alone is significant, if the second orthogonal factor $PC2$ is still statistically significant, then variations in the macroeconomic ambiguity factor contribute to explaining the systematic risk exposures of long/short equity hedge funds. The drawback of this approach is that the estimated coefficients of $PC2$ do not represent macroeconomic ambiguity betas exactly. We report the results of these regressions in Panel A of Table 2.5 for equally weighted and value-weighted portfolios of hedge funds and in Panel B of Table 2.5 for the individual hedge funds. We confirm the statistical significance at the 1% level of the coefficients for both $PC1$ and $PC2$ for the hedge fund portfolios. Furthermore, the percentages of significant coefficients in $PC1$ and $PC2$ are higher for individual hedge funds with high stock market betas (i.e., directional hedge funds).

From this analysis of the ambiguity betas, we can conclude that ambiguity exposure is statistically significant and economically meaningful for directional long/short equity hedge funds. Furthermore, value-weighted portfolios have higher exposure to ambiguity, and hence hedge funds with higher assets under management seem to be more exposed to stock market and macroeconomic ambiguity.

2.4.4 Asset Allocation under Systematic Ambiguity

In this section, we examine the value added by accounting for systematic ambiguity when constructing portfolios of long/short equity hedge funds. We estimate alphas for individual long/short equity hedge funds in our sample under both the ACAPM and the CAPM. These estimated alphas are then used to select hedge funds and construct long-only portfolios whose out-of-sample performance is examined. Portfolios consist of the top 10 hedge funds ranked by their estimated alphas under both models. We assess the risk-adjusted performance of these hedge fund portfolios during the whole sample period (except for the observations from the first estimation window). The portfolio construction method was motivated by the Avramov and Wermers (2006) study that assessed the ex-post out-of-sample performance of various portfolio strategies with monthly rebalancing. The initial estimation window size in that paper is 60 months, and an additional month is added at each realigning point. In our study, we conduct rolling estimation with a fixed-size window of 60 months and rebalance the portfolio every 6 months. The semiannual rebalancing approach corresponds to an average lockup period. As a robustness check, we also provide results with rebalancing frequencies of 1 month and 12 months. The portfolios are constructed on both an equally weighted and a value-weighted basis. We collect the out-of-sample returns of these portfolios (based on the ACAPM and the CAPM) from January 2000 (61st month) to November 2007 (end of the sample period) and compute their realized alphas relative to three benchmark multifactor models: the CAPM, the Fama-French three-factor model, and the Carhart four-factor model.

Table 2.6 reports the risk-adjusted performance or monthly alphas in percentage points of the portfolios of long/short equity hedge funds. The comparison across performance measurement models is organized in columns. EW and VW stand for whether the portfolio is constructed on an equally weighted or value-weighted basis, respectively. If the portfolio construction involves estimating alphas utilizing models with systematic ambiguity, we add the terms *AmbSP* and *AmbIP* for the stock market and macroeconomic ambiguity factors, respectively. We find that the alphas of the portfolios formed with the ambiguity factors are positive, statistically significant, and higher than those estimated for the portfolios formed without the ambiguity factors. For example, with the CAPM performance measurement model over a 12-month holding period, the estimated alpha equals 1.01 for the equally weighted portfolio formed without the ambiguity factor versus 1.23 (1.29) for the models with the stock market (macroeconomic) ambiguity factor. Value-weighted portfolios have higher alphas (except over the 1-month holding period) but similar rankings: for instance, under the CAPM benchmark performance model and over the 12-month holding period, alpha equals 1.09 (with a t-statistic of 1.6) for the no-ambiguity model and increases to 1.54 (with a t-statistic of 2.86) for the *AmbSP* and to 1.92 (with a t-statistic of 3.86) for the $AmbIP$ models. The abnormal performance of all hedge fund portfolios decreases but remains significant over longer and thus more realistic (given hedge funds' lockup periods) holding periods.

Thus, we can conclude from these out-of-sample performance results that selecting long/short equity hedge fund portfolios based on hedge funds' ambiguity-adjusted alphas can generate superior performance that remains statistically and economically significant across all pricing models, even when yearly holding periods are considered.

2.4.5 Robustness Checks

In this section, we discuss the robustness of the empirical results presented so far.

First, in the entire analysis, we relied on two ambiguity factors: the first factor, which assesses the ambiguity surrounding the S&P 500 rate of return forecasts, and the second one, which assesses the ambiguity surrounding the forecasts of Industrial Production Index growth. The results are to a large extent consistent across those two ambiguity factors. We measured ambiguity in both cases as the forecasts' dispersion using the cross-sectional standard deviation. We also studied alternative measures of dispersion such as the range (i.e., the difference between the maximum and minimum forecasted values) and the mean absolute deviation (MAD, computed as the average of the absolute values of the deviations of the individual forecasts from the arithmetic mean). We found consistent results and no significant difference when using those measures as opposed to the standard deviation for the estimation of ambiguity betas and for testing the portfolios' out-of-sample performance.

Second, we always considered equally weighted portfolios as well as value-weighted portfolios to illustrate the impact of large funds. We find that generally the value-weighted portfolios generate superior abnormal performance. Moreover, value-weighted portfolios tend to have higher estimated ambiguity betas regardless of the benchmark risk models. In the portfolio allocation section, our results clearly show that accounting for ambiguity exposure is important for filtering out lower-performing hedge funds.

Third, we vary the estimation window for the portfolio construction: 36 months versus 60 months.[14] We find no significant impact of the rebalancing window on the results.

[14] The results for the 36-month estimation window are not reported for parsimony but are available upon

Fourth, we repeat the analysis while enlarging the scope of performance models to the seven risk factors of the Fung and Hsieh (2004) model that encompasses the asset-based style factors and three option-based trend-following factors. The factors for the Fung and Hsieh (2004) model are available from the D. Hsieh's data library.[15] The first three factors are trend-following risk factors: first, *PTFSBD* is a bond trend-following factor, constructed as the return on a *PTFS*[16] bond lookback straddle; second, *PTFSFX* is a currency trend-following factor, which is constructed as the return on a *PTFS* currency lookback straddle; and finally *PTFSCOM* is a commodity trend-following factor, which is constructed as the return on a *PTFS* commodity lookback straddle. The next two factors are equity-oriented risk factors: the equity market factors are the S&P 500 monthly total return index *SP500TR* and the size spread factor *Size*, defined as the Russell 2000 Index monthly total return minus the S&P 500 monthly total return.[17] The last two factors are bond-oriented risk factors: the bond market factor *Bond* defined as the monthly change in the 10-year Treasury constant maturity yield (month end-to-month end), available from the St. Louis Federal Reserve Economic Data database (FRED); and the credit spread factor *Credit* defined as the monthly change in Moody's Baa yield less 10-year Treasury constant maturity yield (month end-to-month end), also available from FRED.[18]

Table 2.7 reports the correlation coefficients of the stock market and macroeconomic ambiguity factors with the Fung and Hsieh (2004) factors. We find no significant correlation between either ambiguity factor and the Fung and Hsieh (2004) factors that would raise

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request.

[15] Refer to the following website: `https://faculty.fuqua.duke.edu/~dah7/HFRFData.htm`.

[16] *PTFS* stands for primitive trend-following strategy, as described by Fung and Hsieh (2004).

[17] Fung and Hsieh updated the factors provided on the web page. In the original paper they used the Wilshire Small Cap 1750 minus Wilshire Large Cap 750 monthly return.

[18] The two bond-oriented factors from the Fung and Hsieh (2004) model do not represent excess returns on traded assets. Thus, the regression intercept cannot be interpreted as an abnormal return. To obtain abnormal return estimates, one could replace the factors by excess returns on tradable factor-mimicking portfolios: the bond factor would be the return on the US Treasury 10-year government bond index in excess of the risk-free rate; the credit spread factor would be the return on the Baa corporate bond index in excess of the return on treasuries. Since the focus of our analysis is the estimation of ambiguity betas rather than abnormal returns, we use the original Fung and Hsieh (2004) factors.

any multicollinearity issues in the subsequent regression analysis. Table 2.8 reports the factor loadings when using the Fung and Hsieh (2004) model augmented with the ambiguity factors for equally weighted and value-weighted portfolios of long/short equity hedge funds. The total return of the S&P 500 Index and the size spread are the only significant factors. However, the ambiguity factor is always significant, with OLS estimates in the range of 0.11 to 0.17. The explanatory power of the Fung and Hsieh (2004) model measured by the adjusted R^2 coefficient increases noticeably after adding either of the ambiguity factors. Table 2.9 reports ambiguity betas for the individual hedge funds and for hedge funds sorted by their market betas. High market beta hedge funds once again tend to have higher ambiguity exposure estimates, and about 35% of high market beta hedge funds have statistically significant ambiguity exposures versus about 20% for the low market beta hedge funds.

Finally, we analyze the time series properties of ambiguity betas by conducting a rolling regression estimation. To assess the time variation in the ambiguity beta estimates, rolling regressions with a 36-month fixed-size sample window were estimated for the CAPM and Fama-French ambiguity-adjusted models in the case of value-weighted portfolios.[19] At the end of Appendix 2.6.2, Figure 2-2 illustrates rolling ambiguity betas: graphs A and B plot the stock market ambiguity betas, and graphs C and D plot the macroeconomic ambiguity betas. The stock market ambiguity exposures grew during the period after the Long-Term Capital Management (LTCM) collapse, with a peak around 2000 during the dot-com bubble burst. Afterwards, we experienced a strong bull market, and the estimated stock market ambiguity exposures decreased. When we look at the macroeconomic ambiguity exposures, we observe that these betas reached a peak around the turn of the millennium and later increased gradually from 2003 and more rapidly from 2005, contrasting with stock market ambiguity exposures, which are characterized by a negative trend toward the end of the sample period. Thus, long/short equity hedge funds' sensitivity to fundamental

[19]Rolling ambiguity betas for alternative model specifications are available upon request.

uncertainty increased at the same time as their sensitivity to stock market uncertainty decreased.

2.5 Conclusion

Following Maenhout (2004), this study derives the optimal portfolio choice of an equity hedge fund investor who is sensitive to ambiguity. The solution of the asset allocation model reveals that stock market ambiguity induces the investor to reduce his or her allocation to risky assets. Furthermore, imposing market-clearing conditions, we obtain an equilibrium asset-pricing model with stock market ambiguity (ACAPM). In equilibrium, only stock market ambiguity exposure is priced.

In the empirical section, we investigate long/short equity hedge funds and start by estimating their exposures to macroeconomic and stock market ambiguity factors. For this purpose, we measure macroeconomic and stock market ambiguity as the cross-sectional dispersion in the forecasts of Industrial Production Index growth and of S&P 500 Index return from the Livingston Survey, respectively. We construct the ambiguity factors as long/short portfolios of S&P 500 stocks. We estimate ambiguity betas for long/short equity hedge funds and document significant stock market and macroeconomic ambiguity exposures for those funds that follow directional (i.e., high stock market beta) strategies. We then compare the out-of-sample performance of hedge fund portfolios constructed based on their alpha rankings obtained from two pricing models, with and without a systematic ambiguity factor (the ACAPM and the CAPM, respectively). The out-of-sample analysis shows that portfolios constructed based on ACAPM alphas display superior abnormal risk-adjusted returns, especially for the value-weighted portfolios, and that this superior performance is robust to alternative rebalancing horizons and performance measurement models.

This study offers an insight into a thus far neglected dimension of long/short equity hedge fund risk profiles, namely their stock market (or macroeconomic) ambiguity exposures. Our empirical results suggest that for this large category of hedge funds, systematic ambiguity is economically and statistically significant and could be meaningfully exploited by hedge fund investors and fund-of-hedge-fund managers. Interesting extensions of this study would involve examining whether systematic ambiguity matters for understanding other types of hedge fund strategies. Furthermore, it would be useful to determine whether and how ambiguity regarding the skills of individual hedge fund managers should be taken into account during the portfolio construction process. These issues are left for further research.

2.6 Appendices

2.6.1 Optimal Asset Allocation of an Ambiguity-Averse Investor

In this section, we present the mathematical details of the theoretical part of the paper.

Proof of Lemma 2.1

The asset returns under the reference model $\mathcal{P}$ are the following:

$$dB_t = rB_t dt, \tag{2.30}$$

$$\frac{dM_t}{M_t} = \mu_M dt + \sigma_M dZ_t^M, \tag{2.31}$$

$$\frac{dF_t}{F_t} = \mu_F dt + \sigma_F d\widehat{Z}_t, \text{where } d\widehat{Z}_t = \rho dZ_t^M + \sqrt{1-\rho^2} dZ_t^F. \tag{2.32}$$

Let $\mathcal{Q}^H$ be an alternative probability measure parameterized by an appropriately adapted process H_t, with

$$\frac{d\mathcal{Q}^H}{d\mathcal{P}} = \exp\left\{ -\int_0^T \frac{|H_t|^2}{2} dt - \int_0^T H_t dZ_t \right\}, \text{ where } H_t = (h_t^M, h_t^F). \tag{2.33}$$

It follows from equation (2.33) that the relationship between the Brownian motions related to the alternative model and the reference model is linear: $Z_t^H = Z_t + \int_0^t H_s ds$. Hence, $dZ_t^H = dZ_t + H_t dt$. We substitute this formula into equations (2.31) and (2.32) to obtain the stochastic dynamics of asset returns under the alternative probability measure $\mathcal{Q}^H$:

$$\frac{dM_t}{M_t} = \left(\mu_M + \sigma_M h_t^M\right) dt + \sigma_M dZ_t^M, \tag{2.34}$$

$$\frac{dF_t}{F_t} = \left(\mu_F + \rho\sigma_M h_t^M + \sigma_F \sqrt{1-\rho^2} h_t^F\right) dt + \sigma_F \left(\rho dZ_t^M + \sqrt{1-\rho^2} dZ_t^F\right). \tag{2.35}$$

Proof of Lemma 2.2

The formula for the relative entropy is derived as follows:

$$\begin{aligned} Ent(\mathcal{Q}^H|\mathcal{P}) &= \mathbb{E}^H \ln\left(\frac{\mathcal{Q}^H}{\mathcal{P}}\right) = \mathbb{E}^H\left(-\int_0^T \frac{|H_t|^2}{2}dt - \int_0^T H_t dZ_t\right) \\ &= \mathbb{E}^H\left(-\int_0^T \frac{|H_t|^2}{2}dt - \int_0^T H_t(dZ_t^H - H_t dt)\right) \\ &= \mathbb{E}^H \int_0^T \frac{|H_t|^2}{2}dt. \end{aligned} \tag{2.36}$$

Compounding the entropy in equation (2.36), we obtain the value of the global entropy as follows

$$Ent^{glob}(\mathcal{Q}^H|\mathcal{P}) = \delta \int_0^\infty e^{-\delta t}\left(\mathbb{E}^H \int_0^t \frac{(h_s^M)^2 + (h_s^F)^2}{2}ds\right)dt. \tag{2.37}$$

Proof of Theorem 2.1

The expected utility optimization problem has a Max-Min formulation whereby we first minimize with respect to the parameters of the alternative probability laws h^M and h^F searching for the worst-case scenario and then maximize expected discounted utility with respect to the consumption and the portfolio weights:

$$\max_{\theta_M,\theta,C} \min_{h^M,h^F} \mathbb{E}^H \int_0^\infty e^{-\delta t}\left(\frac{C_t^{1-\gamma}}{1-\gamma} + \frac{(h_t^M)^2}{2\psi^M} + \frac{(h_t^F)^2}{2\psi^F}\right)dt. \tag{2.38}$$

The sources of ambiguity h_t^M and h_t^F are assumed to be uncorrelated.

The optimization problem is solved subject to the stochastic wealth dynamics of the investor endowed with initial wealth W_0 as in the standard Merton model. The wealth dynamics under the reference probability measure

$$\begin{aligned} dW_t &= \Big(\big(\theta_M(\mu^M - r) + \theta_F(\mu^F - r) + r\big)W_t - C_t\Big)dt \\ &\quad + W_t(\sigma_M\theta_M + \sigma_F\rho\theta_F)dZ_t^M + W_t\sigma_F\sqrt{1-\rho^2}\theta_F dZ_t^F. \end{aligned} \tag{2.39}$$

The stochastic process for the wealth dynamics distorted by ambiguity satisfies the following:

$$\begin{aligned} dW_t &= \Big(\Big(\theta_M(\mu^M - r + \sigma_M h_t^M) + \theta_F(\mu^F - r + \rho\sigma_M h_t^M + \sigma_F\sqrt{1-\rho^2}h_t^F) + r\Big) W_t \\ &\quad - C_t\Big)dt + W_t\left(\sigma_M\theta_M + \sigma_F\rho\theta^F\right)dZ_t^M + W_t\sigma_F\sqrt{1-\rho^2}\theta_F dZ_t^F. \end{aligned} \tag{2.40}$$

The solution of the optimization problem for the ambiguity-averse agent is achieved with the indirect utility function $J(W,t)$ that should satisfy the HJB equation:

$$\delta J = \max_{C_t,\theta_F,\theta_M}\ \min_{h_t^M,h_t^F}\left\{\frac{C_t^{1-\gamma}}{1-\gamma} + \frac{(h_t^M)^2}{2\psi^M} + \frac{(h_t^F)^2}{2\psi^F} + \mathbb{A}^h(J)\right\}, \tag{2.41}$$

where δ is the investor's time preference and $\mathbb{A}^h(J)$ is a generator as in Merton's intertemporal asset allocation model under the alternative probability measure:

$$\begin{aligned} \mathbb{A}^h(J) &= J_W\Big(\Big(\theta_M(\mu^M - r + \sigma_M h_t^M) + \theta_F(\mu^F - r + \rho\sigma_M h_t^M + \sqrt{1-\rho^2}\sigma_F h_t^F) + r\Big) W_t \\ &\quad - C_t\Big) + \frac{1}{2}J_{WW}W_t^2\Big((\theta_M\sigma_M + \theta_F\sigma_F\rho)^2 + \theta_F^2\sigma_F^2(1-\rho^2)\Big). \end{aligned} \tag{2.42}$$

The specification of the ambiguity aversion coefficient proposed by Maenhout (2004) defines ψ as a function of the indirect utility in the following form:

$$\psi = \frac{\Omega}{WJ_W}, \tag{2.43}$$

where $\Omega = (\Omega^M, \Omega^F)$ is a time-invariant vector proportional to the ambiguity-aversion vector-coefficient $\psi = (\psi^M, \psi^F)$ and J is the indirect utility function. This functional specification will allow us to obtain a closed-form solution to the portfolio choice problem. Note that due to the choice of the CRRA utility function, the previous expression reduces to the following:

$$\psi^M = \frac{\Omega_M}{J(1-\gamma)}, \psi^F = \frac{\Omega_F}{J(1-\gamma)}. \tag{2.44}$$

The minimization problem gives a unique solution for the optimal ambiguity parameters due to the convexity of the function with respect to h_t^M and h_t^F:

$$h_t^M = -\Omega_M \sigma_M(\theta_M + \theta_F \rho),\ h_t^F = -\Omega_F \theta_F \sigma_F \sqrt{1-\rho^2}. \tag{2.45}$$

After solving the minimization part, we substitute the expressions for h_t^M and h_t^F into the objective function and solve the maximization problem in order to determine the optimal portfolio weights θ_M and θ_F and optimal consumption C^*.

Using the first-order condition of the HJB equation with respect to C, optimal consumption can be obtained in terms of indirect utility as follows:

$$C^* = [J_W]^{-\frac{1}{\gamma}}. \tag{2.46}$$

The implicit expressions for the optimal fraction of wealth invested in the stock market and in the hedge fund respectively are the following:

$$\theta_M = \frac{\mu^M - r - \theta_F \rho \sigma_M(\Omega_M \sigma_M + \gamma \sigma_F)}{(\gamma + \Omega_M)\sigma_M^2}, \tag{2.47}$$

$$\theta_F = \frac{\mu^F - r - \theta_M \rho \sigma_M(\Omega_M \sigma_M + \gamma \sigma_F)}{\gamma \sigma_F^2 + \Omega_M \rho^2 \sigma_M^2 + \Omega_F \sigma_F^2 (1-\rho^2)}. \tag{2.48}$$

Proof of Proposition 2.1

The explicit formulas for the optimal allocation are derived in several steps. First, consider the equations (2.47) and (2.48) as a system of linear equations:

$$A\theta_M + B\theta_F = \mu^M - r, \tag{2.49}$$

$$C\theta_M + D\theta_F = \mu^F - r, \tag{2.50}$$

or in matrix notation:

$$M\theta = \mu - r, \tag{2.51}$$

where

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}, \theta = \begin{pmatrix} \theta_M \\ \theta_F \end{pmatrix}, \text{ and } \mu - r = \begin{pmatrix} \mu^M - r \\ \mu^F - r \end{pmatrix}. \tag{2.52}$$

The coefficients A, B, C, and D are functions of volatilities σ_M and σ_F, ambiguities Ω_M and Ω_F, and correlation ρ:

$$A = (\gamma + \Omega_M)\sigma_M^2, \tag{2.53}$$

$$B = \rho\sigma_M(\Omega_M\sigma_M + \gamma\sigma_F), \tag{2.54}$$

$$C = B, \tag{2.55}$$

$$D = \gamma\sigma_F^2 + \Omega_M\rho^2\sigma_M^2 + \Omega_F\sigma_F^2(1 - \rho^2). \tag{2.56}$$

The explicit solution in terms of an inverse matrix[20] is the following:

$$\theta = M^{-1}(\mu - r), \tag{2.57}$$

[20] Assuming an inverse matrix exists, i.e., the determinant is not zero: $AD \neq BC$.

or in scalar notation:

$$\theta_M = \frac{D(\mu^M - r) - B(\mu^F - r)}{AD - BC}, \tag{2.58}$$

$$\theta_F = \frac{A(\mu^F - r) - C(\mu^M - r)}{AD - BC}. \tag{2.59}$$

The above equations are equivalent to the following:

$$\theta_M = \frac{(\mu^M - r) - \frac{B}{D}(\mu^F - r)}{A - \frac{B^2}{D}}, \tag{2.60}$$

$$\theta_F = \frac{(\mu^F - r) - \frac{C}{A}(\mu^M - r)}{D - \frac{B^2}{A}}. \tag{2.61}$$

The optimal portfolio weights for both the stock market and the hedge fund are proportional to the expected excess return of the corresponding asset and inversely proportional to risk and ambiguity as contained in the denominators $A - \frac{B^2}{D}$ and $D - \frac{B^2}{A}$. Moreover, the optimal portfolio weight for each asset is negatively related to the expected excess return of the other asset adjusted by the relative beta $\beta_{M,F} = \frac{B}{D}$ or $\beta_{F,M} = \frac{C}{A}$. The higher the expected excess return of the other asset, the lower the allocation to the specific asset.

Substituting the values of coefficients A, B, C, and D into equations (2.60) and (2.61), the general explicit formulas for the optimal portfolio weights are as follows:

$$\theta_M = \frac{\mu^M - r - \frac{\rho\sigma_M(\Omega_M\sigma_M + \gamma\sigma_F)}{\gamma\sigma_F^2 + \Omega_M\rho^2\sigma_M^2 + \Omega_F\sigma_F^2(1-\rho^2)}\left(\mu^F - r\right)}{(\gamma + \Omega_M)\sigma_M^2 - \frac{\rho^2\sigma_M^2(\Omega_M\sigma_M + \gamma\sigma_F)^2}{\gamma\sigma_F^2 + \Omega_M\rho^2\sigma_M^2 + \Omega_F\sigma_F^2(1-\rho^2)}}, \tag{2.62}$$

$$\theta_F = \frac{\mu^F - r - \frac{\rho\sigma_M(\Omega_M\sigma_M + \gamma\sigma_F)}{(\gamma+\Omega_M)\sigma_M^2}\left(\mu^M - r\right)}{\gamma\sigma_F^2 + \Omega_M\rho^2\sigma_M^2 + \Omega_F\sigma_F^2(1-\rho^2) - \frac{\rho^2\sigma_M^2(\Omega_M\sigma_M + \gamma\sigma_F)^2}{(\gamma+\Omega_M)\sigma_M^2}}. \tag{2.63}$$

The influence of systematic stock market ambiguity and idiosyncratic hedge fund ambiguity on the optimal portfolio choice depends on the correlation coefficient ρ. On the one hand, when ρ approaches 1, $\Omega_F = 0$ in the formula for coefficient D and the impact of hedge fund ambiguity disappears. Stock market ambiguity affects the optimal portfolio weights either by increasing overall risk aversion (denominator effect) or through the relative betas $\beta_{M,F} = \frac{B}{D}$ and $\beta_{F,M} = \frac{C}{A}$. On the other hand, when ρ approaches 0, the relative betas are equal to zero (there is no correlation between hedge fund and stock market returns: $\beta_{M,F} = \beta_{F,M} = 0$), $B = C = 0$, and the formulas for the optimal allocation reduce to the following expressions:

$$\theta_M = \frac{\mu^M - r}{(\gamma + \Omega_M)\sigma_M^2}, \tag{2.64}$$

$$\theta_F = \frac{\mu^F - r}{(\gamma + \Omega_F)\sigma_F^2}. \tag{2.65}$$

The equations (2.64) and (2.65) are the Merton optimal portfolio weights in the case of ambiguity-averse preferences. Ambiguity amplifies the impact of risk aversion, reducing the optimal portfolio allocation to risky securities.

In the case of no ambiguity ($\Omega_M = 0$ and $\Omega_F = 0$), the optimal portfolio choice coincides with the standard Merton optimal portfolio weights for two risky assets and a risk-averse investor. In this case, we have $A = \gamma\sigma_M^2$, $B = C = \rho\gamma\sigma_M\sigma_F$, and $D = \gamma\sigma_F^2$, and the portfolio weights are the following:

$$\theta_M = \frac{\mu^M - r - \beta_{M,F}(\mu^F - r)}{\gamma\sigma_M^2(1 - \rho^2)}, \tag{2.66}$$

$$\theta_F = \frac{\mu^F - r - \beta_{F,M}(\mu^M - r)}{\gamma\sigma_F^2(1 - \rho^2)}, \tag{2.67}$$

where

$$\beta_{M,F} = \frac{\rho\sigma_M}{\sigma_F}, \tag{2.68}$$

$$\beta_{F,M} = \frac{\rho\sigma_F}{\sigma_M}. \tag{2.69}$$

Proof of Proposition 2.2

Using the formula for the optimal portfolio weights in Theorem 2.1 and the market-clearing conditions (2.26), we obtain:

$$\mu^M - r = (\gamma + \Omega_M)\sigma_M^2, \tag{2.70}$$

$$\mu^F - r = \rho\sigma_M(\Omega_M\sigma_M + \gamma\sigma_F), \tag{2.71}$$

which is equivalent to

$$\mu^M = r + \gamma\sigma_M^2 + \Omega_M\sigma_M^2, \tag{2.72}$$

and for the hedge fund's expected returns,

$$\mu^F = r + \gamma\sigma_F\rho\sigma_M + \Omega_M\rho\sigma_M^2. \tag{2.73}$$

2.6.2 Tables and Figures

Table 2.1: Descriptive statistics. Panel A reports hedge fund characteristics for each year as well as over the whole sample period from January 1994 to November 2007. *Sample Size* is the average number of live hedge funds. *Fees* are management fee/incentive fee in percentage points. *Lockup* is the average lockup period in months. *Leverage* is the average hedge fund leverage level as a percentage of AUM. *NumbLev* is the number of hedge funds that report nonzero leverage. AUM is the average AUM in USD mn. The average (median) life of hedge funds in the sample is 60 (50) months. The total number of both live and defunct hedge funds in the sample is 2070. Panel B reports descriptive statistics on hedge fund returns: cross-sectional mean, median, and standard deviation of annualized hedge fund returns, annualized standard deviation, annualized Sharpe ratio, annual skewness and kurtosis. For the Jacques-Bera test we report the sample descriptive statistics of the indicator JB ($JB = 0$ if the Null that the return distribution is Gaussian cannot be rejected vs. $JB = 1$ if the Null is rejected at 5% significance level), the JB p-value and the value of the JB statistic.

Panel A

Year	***Sample Size***	***Fees***	***LockUp***	***Leverage***	***NumbLev***	***AUM***
1994	206	1.09 / 17.00	2.84	127	54	58
1995	265	1.11 / 17.58	2.92	126	70	49
1996	358	1.11 / 18.04	3.17	127	95	53
1997	462	1.11 / 18.44	3.40	131	137	57
1998	555	1.12 / 18.64	3.55	138	178	58
1999	653	1.13 / 18.76	4.05	140	229	56
2000	778	1.15 / 18.97	4.60	141	287	77
2001	886	1.18 / 19.10	5.12	138	306	78
2002	959	1.21 / 19.08	5.27	137	312	77
2003	1014	1.24 / 19.00	5.34	135	305	74
2004	1101	1.28 / 19.08	5.41	135	310	92
2005	1168	1.33 / 19.12	5.46	133	310	107
2006	1143	1.36 / 19.00	5.54	133	296	130
2007	986	1.38 / 18.91	5.41	133	247	152
All years	752	1.20 / 18.60	4.43	134	224	80

Panel B

	Mean	**Median**	**Std Dev**	**Max**	**Min**
Return	12.90	11.74	13.19	136.29	−54.46
Std Dev	15.92	12.76	11.77	112.84	0.08
Sharpe	0.70	0.68	0.84	5.89	−5.46
Skewness	0.76	0.72	0.61	3.21	−1.40
Kurtosis	4.25	3.56	1.58	15.70	2.10
JB	0.66	1.00	0.47	1.00	0.00
***JB p*-value**	0.07	0.01	0.13	0.50	0.00
***JB*-stat**	49.13	14.83	68.12	666.83	0.06

Table 2.2: Panel A reports pairwise Pearson, Kendall, and Spearman correlation coefficients between ambiguity factors and the excess market return (*EMKT*), the Fama-French factors (*SMB* and *HML*), and momentum factor (*MOM*). Panel B reports the Granger causality test for the momentum factor and the ambiguity factors. Granger causality is tested with a maximum of 5 lags (the actual number of lags is chosen using the Bayesian information criterion, *BIC*) and a 5% significance level. F and G are values of the F-statistic of the Granger test. Fc and Gc are critical values from the F-distribution. Rule: if $F > Fc$, then we reject the null hypothesis that momentum does not cause the ambiguity factor; if $G > Gc$, then we reject the null hypothesis that the ambiguity factor does not cause momentum. *AmbIP* is the macroeconomic ambiguity factor based on the Industrial Production Index growth forecasts. *AmbSP* is the stock market ambiguity factor based on the S&P 500 return forecasts.

Panel A		***AmbSP***			***AmbIP***	
	Pearson	**Kendall**	**Spearman**	**Pearson**	**Kendall**	**Spearman**
EMKT	0.07	0.03	0.05	−0.11	−0.09	−0.13
SMB	0.37	0.08	0.13	0.26	0.07	0.10
HML	−0.37	−0.12	−0.17	−0.16	0.02	0.02
MOM	0.17	0.05	0.07	0.65	0.31	0.43

Panel B	***AmbSP***	***AmbIP***
F	1.18	8.25
Fc	3.90	3.90
G	1.49	1.72
Gc	3.90	3.90

Table 2.3: Multifactor model for the hedge fund portfolios. This table presents the results of OLS regressions of hedge fund portfolio returns for the CAPM, Fama-French, and Carhart model specifications. *EW* denotes the equally weighted portfolio of hedge funds, and *VW* the value-weighted portfolio of hedge funds. *AmbIP* is the macroeconomic ambiguity factor based on the Industrial Production Index growth forecasts. *AmbSP* is the stock market ambiguity factor based on the S&P 500 return forecasts. 1% (5%) statistical significance is indicated by two (one) asterisks. The t-statistics of regression coefficients are reported in parentheses.

CAPM	***EW***	***VW***	***EW,AmbSP***	***VW,AmbSP***	***EW,AmbIP***	***VW,AmbIP***
Alpha	0.62**	0.60**	0.67**	0.67**	0.62**	0.59**
	(5.36)	(3.77)	(6.00)	(4.43)	(5.76)	(4.20)
EMKT	0.51**	0.49**	0.51**	0.48**	0.53**	0.52**
	(18.73)	(13.05)	(19.19)	(13.38)	(20.64)	(15.29)
Amb	—	—	0.07**	0.11**	0.09**	0.15**
	—	—	(3.91)	(4.34)	(5.24)	(6.63)
Adj R^2	0.68	0.50	0.70	0.55	0.72	0.61

FF	***EW***	***VW***	***EW,AmbSP***	***VW,AmbSP***	***EW,AmbIP***	***VW,AmbIP***
Alpha	0.62**	0.62**	0.63**	0.63**	0.61**	0.60**
	(7.18)	(4.90)	(7.21)	(5.00)	(7.30)	(5.08)
EMKT	0.46**	0.41**	0.46**	0.42**	0.48**	0.45**
	(19.61)	(11.89)	(19.50)	(12.00)	(20.50)	(13.53)
SMB	0.26**	0.31**	0.26**	0.30**	0.25**	0.28**
	(10.64)	(8.65)	(10.17)	(8.12)	(10.06)	(8.05)
HML	−0.01	−0.05	0.00	−0.03	0.01	−0.02
	(−0.23)	(−1.05)	(−0.02)	(−0.62)	(0.24)	(−0.41)
Amb	—	—	0.01	0.03	0.05**	0.10**
	—	—	(0.76)	(1.42)	(3.42)	(5.02)
Adj R^2	0.83	0.70	0.83	0.70	0.84	0.74

Carhart	***EW***	***VW***	***EW, AmbSP***	***VW, AmbSP***	***EW, AmbIP***	***VW, AmbIP***
Alpha	0.53**	0.45**	0.54**	0.45**	0.54**	0.46**
	(6.48)	(3.98)	(6.47)	(4.04)	(6.48)	(4.07)
EMKT	0.49**	0.47**	0.49**	0.47**	0.49**	0.47**
	(21.67)	(15.29)	(21.51)	(15.29)	(21.61)	(15.32)
SMB	0.25**	0.28**	0.24**	0.27**	0.24**	0.27**
	(10.52)	(8.76)	(10.18)	(8.35)	(10.32)	(8.53)
HML	0.01	−0.01	0.01	0.00	0.01	−0.01
	(0.34)	(−0.34)	(0.41)	(−0.08)	(0.38)	(−0.25)
MOM	0.08**	0.17**	0.08**	0.17**	0.08**	0.15**
	(5.11)	(7.55)	(5.04)	(7.43)	(3.70)	(5.34)
Amb	—	—	0.01	0.02	0.01	0.02
	—	—	(0.33)	(0.93)	(0.53)	(1.00)
Adj R^2	0.85	0.77	0.85	0.77	0.85	0.77

Table 2.4: Ambiguity betas and alphas relative to CAPM, Fama-French, and Carhart models for individual long/short equity hedge funds. *AmbSP* is the stock market ambiguity. *AmbIP* is the macroeconomic ambiguity. Results are reported for the whole sample as well as for the subsample of high market beta hedge funds (those for which the market beta is statistically significant at the 10% level and its absolute value is higher than 0.10) and low market beta hedge funds (the rest of the sample). The number of high market beta hedge funds is 838 out of a total of 2070. %ofSgnf denotes the percentage of significant estimates at the 10% level. #ofSgnf denotes the number of significant estimates at the 10% level.

All Hedge Funds		***AmbSP***			***AmbIP***	
	CAPM	**FF**	**Carhart**	**CAPM**	**FF**	**Carhart**
Mean *Alpha*	0.99	0.87	0.75	1.09	1.00	1.11
Median *Alpha*	0.87	0.81	0.75	0.90	0.90	0.95
%ofSgnf *Alpha*	47.58	44.09	41.52	47.26	45.94	40.80
Mean *Beta*	0.11	−0.02	−0.01	0.13	0.08	0.08
Median *Beta*	0.13	−0.08	−0.07	0.15	0.12	0.10
%ofSgnf *Beta*	26.91	15.12	13.96	31.79	24.40	14.44
Mean *Adj* R^2	0.35	0.39	0.44	0.33	0.41	0.39
Median *Adj* R^2	0.33	0.39	0.46	0.32	0.42	0.38

Low Market Beta		***AmbSP***			***AmbIP***	
	CAPM	**FF**	**Carhart**	**CAPM**	**FF**	**Carhart**
Mean *Alpha*	0.63	0.56	0.45	1.26	1.01	1.11
Median *Alpha*	0.80	0.72	0.69	0.82	0.97	0.86
%ofSgnf *Alpha*	48.65	50.62	50.65	43.75	44.74	40.48
Mean *Beta*	0.05	−0.03	−0.01	0.12	0.13	0.13
Median *Beta*	−0.06	−0.09	−0.06	0.14	0.14	0.13
%ofSgnf *Beta*	17.37	12.94	12.54	22.54	18.21	13.68
#ofSgnf *Beta*	111.00	81.00	77.00	144.00	114.00	84.00
Mean *Adj* R^2	0.17	0.21	0.27	0.16	0.22	0.22
Median *Adj* R^2	0.14	0.16	0.21	0.11	0.14	0.16

High Market Beta		***AmbSP***			***AmbIP***	
	CAPM	**FF**	**Carhart**	**CAPM**	**FF**	**Carhart**
Mean *Alpha*	1.09	0.99	0.90	1.05	1.00	1.11
Median *Alpha*	0.91	0.92	0.89	0.90	0.90	0.96
%ofSgnf *Alpha*	47.31	41.81	38.21	48.25	46.29	40.93
Mean *Beta*	0.13	−0.01	−0.01	0.13	0.07	0.06
Median *Beta*	0.15	−0.08	−0.07	0.16	0.11	0.08
%ofSgnf *Beta*	31.68	16.54	15.13	36.51	27.87	15.35
#ofSgnf *Beta*	446.00	232.00	212.00	514.00	391.00	215.00
Mean *Adj* R^2	0.40	0.46	0.50	0.38	0.47	0.46
Median *Adj* R^2	0.38	0.44	0.51	0.37	0.46	0.45

Table 2.5: Carhart multifactor model with the macroeconomic ambiguity factor adjusted for collinearity with the momentum factor (original *MOM* and *AmbIP* are substituted by their two orthogonal principal components *PC*1 and *PC*2). All other variables are identical to the previous tables. Panel A reports estimated coefficients for the hedge fund portfolios (equally weighted *EW* and value-weighted *VW*). Panel B reports estimated coefficients of individual hedge funds for the whole sample, low market beta hedge funds, and high market beta hedge funds. 1% (5%) statistical significance is indicated by two (one) asterisks. The t-statistics of regression coefficients are reported in parentheses. %ofSgnf denotes the percentage of significant estimates at the 10% level. #ofSgnf denotes the number of significant estimates at the 10% level.

Panel A	***EW***	***VW***	***EW***	***VW***
Alpha	0.60**	0.58**	0.60**	0.58**
	(7.33)	(5.13)	(7.40)	(5.24)
EMKT	0.49**	0.46**	0.49**	0.47**
	(21.15)	(14.58)	(21.61)	(15.32)
SMB	0.24**	0.27**	0.24**	0.27**
	(10.09)	(8.16)	(10.32)	(8.53)
HML	0.01	−0.01	0.01	−0.01
	(0.40)	(−0.22)	(0.38)	(−0.25)
MOM	0.05**	0.10**	—	—
	(4.44)	(6.58)	—	—
***PC*1**	—	—	0.05**	0.10**
	—	—	(4.52)	(6.82)
***PC*2**	—	—	0.06**	0.11**
	—	—	(2.45)	(3.44)
Adj R^2	0.84	0.76	0.85	0.77

Panel B	**All Hedge Funds**	**Low Market Beta**	**High Market Beta**
Mean *Alpha*	1.15	1.56	0.99
Median *Alpha*	0.87	1.24	0.82
%ofSgnf *Alpha*	48.51	54.23	46.62
Mean *PC*1	0.20	0.27	0.19
Median *PC*1	0.17	0.18	0.16
%ofSgnf *PC*1	14.30	10.11	19.42
#ofSgnf *PC*1	296.00	57.00	239.00
Mean *PC*2	0.20	0.23	0.19
Median *PC*2	0.22	0.25	0.21
%ofSgnf *PC*2	27.58	23.13	30.62
#ofSgnf *PC*2	571.00	142.00	429.00
Mean *Adj* R^2	0.43	0.27	0.48
Median *Adj* R^2	0.42	0.23	0.47

Table 2.6: Comparative performance analysis of the portfolios formed with and without the ambiguity factor. The portfolios consist of 10 equally weighted or value-weighted hedge funds with the highest alpha ranking. The ranking is based on the historical alphas estimated by CAPM (one-factor model) versus ACAPM (two-factor model with the ambiguity factor: either the stock market ambiguity *AmbSP* or macroeconomic ambiguity *AmbIP*). The estimation window is 60 months. The out-of-sample portfolio formation period is 1 month, 6 months, or 12 months such that we rebalance the portfolio every month, every 6 months, or every 12 months. The table reports risk-adjusted portfolio performance, i.e. estimated alphas relative to CAPM, Fama-French (FF), and Carhart models. 1% (5%) statistical significance is marked by two (one) asterisks. The t-statistics of regression coefficients appear in parentheses.

	1 month			**6 months**		
Portfolio	**CAPM**	**FF**	**Carhart**	**CAPM**	**FF**	**Carhart**
EW Alpha	2.21**	1.98**	1.93**	1.44**	1.22**	1.16**
	(4.6)	(4.72)	(4.56)	(3.4)	(3.57)	(3.38)
EW, AmbSP Alpha	2.43**	2.25**	2.23**	1.56**	1.3**	1.29**
	(5.72)	(5.5)	(5.39)	(3.92)	(3.73)	(3.66)
EW, AmbIP Alpha	1.92***	1.76**	1.77**	1.69**	1.46**	1.47**
	(4.89)	(4.41)	(4.38)	(4.86)	(4.32)	(4.29)
VW Alpha	2.09**	1.71**	1.67**	1.47*	1.14*	1.00
	(3.15)	(2.8)	(2.7)	(2.15)	(2.14)	(1.88)
VW, AmbSP Alpha	2.26**	2.04**	1.97**	1.69**	1.42**	1.39**
	(3.28)	(3.07)	(2.93)	(3.1)	(2.75)	(2.67)
VW, AmbIP Alpha	1.83**	1.58**	1.5**	1.94**	1.67**	1.66**
	(3.32)	(2.97)	(2.81)	(3.92)	(3.5)	(3.45)

	12 months		
Portfolio	**CAPM**	**FF**	**Carhart**
EW Alpha	1.01*	0.85**	0.78*
	(2.52)	(2.7)	(2.47)
EW, AmbSP Alpha	1.23**	1.04**	1.04**
	(3.3)	(3.23)	(3.18)
EW, AmbIP Alpha	1.29**	1.05**	1.08**
	(3.99)	(3.3)	(3.35)
VW Alpha	1.09	1.07*	0.85
	(1.6)	(2.06)	(1.69)
VW, AmbSP Alpha	1.54**	1.52**	1.45**
	(2.86)	(2.91)	(2.76)
VW, AmbIP Alpha	1.92**	1.8**	1.77**
	(3.86)	(3.64)	(3.54)

Table 2.7: Robustness check: pairwise Pearson, Kendall, and Spearman correlation coefficients between ambiguity factors and Fung-Hsieh hedge fund factors. *AmbIP* is the macroeconomic ambiguity factor based on Industrial Production Index growth forecasts. *AmbSP* is the stock market ambiguity factor based on S&P 500 return forecasts. *PTFSBD* is a bond trend-following factor, *PTFSFX* is a currency trend-following factor, *PTFSCOM* is a commodity trend-following factor, *SP500TR* is the S&P 500 monthly total return index, *Size* is defined as the Russell 2000 Index monthly total return minus the S&P 500 monthly total return, *Bond* is defined as the monthly change in the 10-year Treasury constant maturity yield (month end-to-month end), and *Credit* is defined as the monthly change in Moody's Baa yield less 10-year Treasury constant maturity yield (month end-to-month end).

	AmbSP			***AmbIP***		
FH factors	**Pearson**	**Kendall**	**Spearman**	**Pearson**	**Kendall**	**Spearman**
PTFSBD	0.14	0.12	0.17	−0.02	−0.01	−0.02
PTFSFX	−0.12	−0.15	−0.21	0.00	−0.01	−0.02
PTFSCOM	0.07	0.02	0.04	0.19	0.11	0.16
SP500TR	−0.01	0.01	0.02	−0.19	−0.12	−0.17
Size	−0.01	0.02	0.02	0.02	−0.04	−0.04
Bond	−0.01	0.02	0.03	0.04	0.04	0.06
Credit	−0.11	−0.07	−0.10	−0.11	0.01	0.01

Table 2.8: Robustness check: OLS regression of hedge fund portfolio returns on the Fung-Hsieh factors. *EW* denotes the equally weighted hedge fund portfolio, *VW* denotes the value-weighted hedge fund portfolio. *AmbSP* is the ambiguity factor based on the S&P 500 Index return forecasts. *AmbIP* is the macroeconomic ambiguity factor based on Industrial Production Index growth forecasts. 1% (5%) statistical significance is indicated by two (one) asterisks. The t-statistics of regression coefficients appear in parentheses.

	EW	***VW***	***EW,AmbSP***	***VW,AmbSP***	***EW,AmbIP***	***VW,AmbIP***
Const	0.29	0.39	0.01	0.03	−0.03	−0.09
	(0.53)	(0.56)	(0.02)	(0.04)	(−0.06)	(−0.14)
PTFSBD	−1.19	−2.03	−1.96*	−3.03*	−0.90	−1.60
	(−1.20)	(−1.61)	(−2.09)	(−2.57)	(−0.98)	(−1.41)
PTFSFX	1.27	1.15	1.93*	2.00*	1.49*	1.47
	(1.59)	(1.13)	(2.55)	(2.11)	(2.00)	(1.61)
PTFSCOM	0.86	1.87	0.44	1.32	−0.14	0.39
	(0.77)	(1.32)	(0.43)	(1.01)	(−0.13)	(0.30)
SP500TR	0.46**	0.42**	0.47**	0.42**	0.50**	0.47**
	(12.88)	(9.19)	(13.96)	(10.05)	(14.67)	(11.32)
Size	−0.01	−0.01*	0.00	−0.01	−0.01	−0.01*
	(−1.56)	(−2.10)	(−1.37)	(−1.94)	(−1.47)	(−2.07)
Bond	0.29	−0.19	0.34	−0.13	0.15	−0.40
	(0.45)	(−0.24)	(0.57)	(−0.18)	(0.25)	(−0.56)
Credit	0.02	−0.14	0.22	0.11	0.19	0.11
	(0.08)	(−0.38)	(0.81)	(0.34)	(0.72)	(0.34)
Amb	—	—	0.12**	0.16**	0.11**	0.17**
	—	—	(5.15)	(5.33)	(5.21)	(6.29)
Adj R^2	0.50	0.35	0.57	0.45	0.57	0.48

Table 2.9: Robustness check: ambiguity betas and regression intercepts relative to the Fung-Hsieh multifactor model for individual long/short equity hedge funds. *AmbSP* is the stock market ambiguity. *AmbIP* is the macroeconomic ambiguity. Results (mean, median, and percentage of significant estimates at the 10% level) are reported for the whole sample as well as for high market beta hedge funds (those for which the market beta is statistically significant at the 10% level and its absolute value is higher than 0.10) and low market beta hedge funds (the rest of the sample). The number of high market beta hedge funds is 838 out of a total of 2070. %ofSgnf denotes the percentage of significant estimates at the 10% level. #ofSgnf denotes the number of significant estimates at the 10% level.

AmbSP	**All Hedge Funds**	**Low Market Beta**	**High Market Beta**
Mean *Const*	−2.36	−9.99	−0.99
Median *Const*	−1.93	−1.91	−1.96
%ofSgnf *Const*	21.77	31.62	19.56
Mean *Beta*	0.17	0.13	0.19
Median *Beta*	0.16	−0.08	0.19
%ofSgnf *Beta*	29.52	19.90	35.63
Mean *Adj* R^2	0.34	0.24	0.36
Median *Adj* R^2	0.32	0.18	0.34

AmbIP	**All Hedge Funds**	**Low Market Beta**	**High Market Beta**
Mean *Const*	−3.32	−3.62	−3.22
Median *Const*	−2.26	0.73	−2.95
%ofSgnf *Const*	17.11	22.41	15.82
Mean *Beta*	0.21	0.18	0.23
Median *Beta*	0.19	0.17	0.20
%ofSgnf *Beta*	29.37	19.73	35.42
Mean *Adj* R^2	0.32	0.22	0.35
Median *Adj* R^2	0.31	0.16	0.33

Figure 2-1: Time series of ambiguity factor returns. Graph A shows monthly returns for the stock market ambiguity factor, graph B returns for the macroeconomic ambiguity factor. Stock market ambiguity is measured by the standard deviation of the forecasts of S&P 500 Index returns. Macroeconomic ambiguity is measured by the standard deviation of the forecasts of Industrial Production Index growth. The factor return is defined as the out-of-sample return of a long/short equally weighted portfolio of stocks where long positions are taken in the top decile and short positions are taken in the bottom decile of stocks ranked by their ambiguity sensitivities.

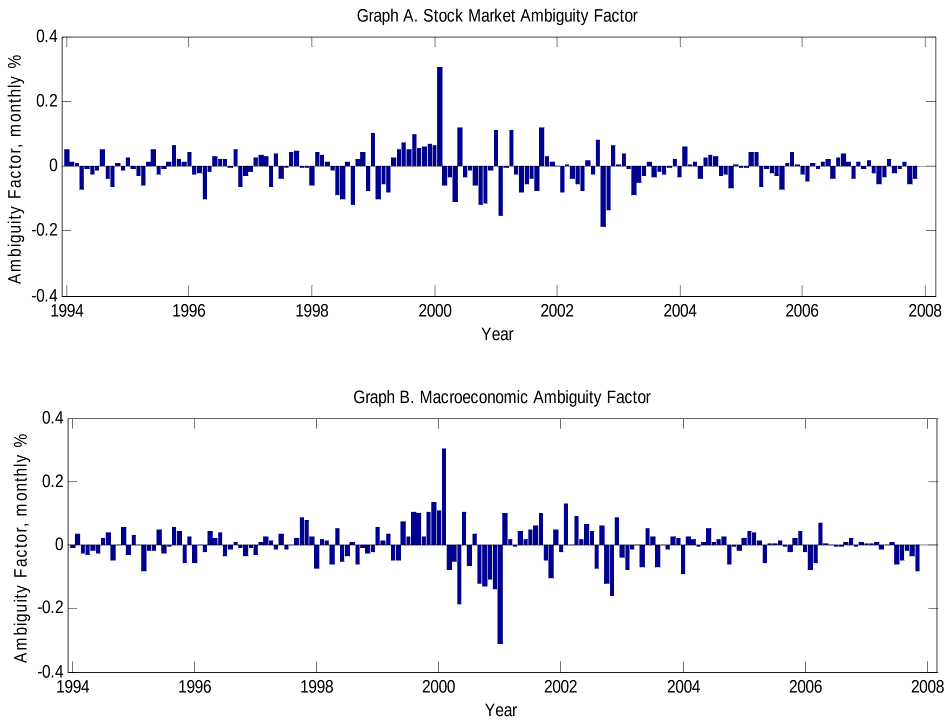
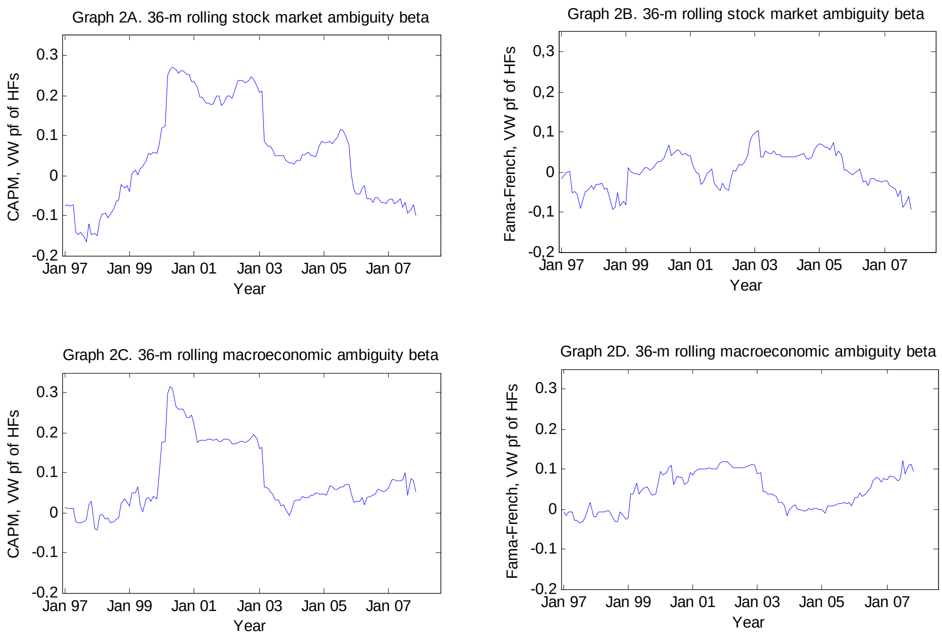


Figure 2-2: Time series properties of ambiguity betas. The CAPM (graphs A and C) and Fama-French (graphs B and D) ambiguity-adjusted models are estimated for the value-weighted portfolio of hedge funds by OLS rolling regressions with a 36-month fixed-size sample window. Graphs A and B plot the stock market ambiguity betas based on S&P 500 return forecasts. Graphs C and D plot the macroeconomic ambiguity betas based on Industrial Production Index growth forecasts.



Chapter 3

Hedge Fund Price Pressure in Convertible Bond Markets

Abstract

Prices of convertible bonds are sensitive to unexpected demand shocks generated by convertible arbitrage hedge funds. This paper investigates whether the price pressure created by innovations in hedge fund demand can account for the systematic mispricing of convertible bonds. We empirically construct the risk factor associated with hedge fund price pressure and document the nonnegligible risk premium embedded in convertible bond returns. Moreover, we demonstrate that contemporaneous returns of convertible bond mutual funds have significant negative exposure to the price-pressure factor. Price-pressure risk is amplified during financial crises, creating the risk of a convertible bond sell-off.

JEL codes: G12

Keywords: Convertible Bonds, Convertible Arbitrage, Hedge Funds, Price Pressure.

3.1 Introduction

A convertible bond is a fixed-income security that gives its holders an option to convert the bond into a predetermined number of shares of the issuing firm. The price of a plain vanilla convertible bond can, in principle, be modeled as the price of a corporate bond plus an embedded option. However, the structure and the embedded clauses of convertible bonds complicate the actual pricing task. The systematic underpricing of convertible bonds — the difference between the market price and the theoretical value — has been well documented in the literature.[1] The attractive valuation of convertible bonds provides a basis to pursue a convertible arbitrage strategy. The plain vanilla convertible arbitrage strategy consists of a long position in the convertible bond and a short position in the underlying stock. Traditionally, major holders of convertible bonds have been specialized mutual funds, so-called convertible bond mutual funds, or other long-only investors with a similar investment mandate. However, along with the growth of the hedge fund industry since the 1990s, the focus of convertible bond investing has shifted toward arbitrage. According to the estimates reported by Mitchell et al. (2007), about 75% of all convertible bond issues were used by hedge funds in 2005. Moreover, Choi et al. (2009) report that the recent growth in trading volume and new issues of convertible bonds has been primarily driven by demand from convertible arbitrage hedge funds. Based on the academic evidence and common beliefs circulating among brokers, sell-side research, and the press, we postulate that the major source of demand for convertible bonds is convertible arbitrage hedge funds, at least in the last decade. This sector's homogenous demand structure with the dominant role of arbitrageurs creates a possibility of price-pressure risk that could have implications for the pricing of convertible bonds.

This paper proposes to explain the underpricing of convertible bonds by the existence of systematic price-pressure risk, originating from innovations in hedge fund demand. We

[1] See Section 3.2 for a literature review.

hypothesize that the mispricing of convertible bonds disappears after taking into account price-pressure risk. In this paper, in order to avoid the complexity of computing theoretical prices of individual convertible bonds having a large variety of contractual specifications, we do not evaluate the mispricing of convertible bonds as a discrepancy between the market price and the theoretical price. To tackle the underpricing anomaly, we instead investigate the space of convertible bond risk factors empirically. We consider the puzzle of convertible bond underpricing in the context of a missing risk factor in a risk-based factor model. The unspecified risk factors produce modeling errors in any pricing model. Fully accounting for all potential risk factors is paramount in reducing mispricing of convertible bonds.

The paper constructs the convertible bond risk factor associated with the price pressure of convertible arbitrage hedge funds and computes the corresponding risk premium that an investor would demand to compensate for price-pressure risk. Furthermore, the paper empirically tests the hypothesis that augmenting the conventional convertible bond multifactor model by the price-pressure risk factor improves the explanatory power of the risk model.

Convertible arbitrage hedge funds are liquidity suppliers to the convertible bond market. Hence the price-pressure risk factor can be viewed as another liquidity factor. Securities with high liquidity sensitivity are traded at a discount relative to those with low sensitivity. Likewise, convertible bonds with high sensitivities to hedge fund demand shocks carry a price discount and hence possess higher expected returns as compensation for price-pressure risk. As the price pressure amplifies during financial crises, an investor would prefer to hold securities that are unlikely to be liquidated by convertible arbitrage hedge funds, namely convertible bonds with low or negative sensitivity to the hedge fund demand innovation. Therefore these convertible bonds embed a premium in contemporaneous prices and have low expected returns. The spread in expected returns of convertible bonds with high and low sensitivities to aggregate demand shocks defines the price-pressure risk factor.

This paper contributes to our understanding of the convertible bond market's fragility due to price pressure and potential forced-selling risk. The convertible arbitrage hedge fund strategy is proven to be vulnerable to exogenous shocks, particularly during market-wide liquidity squeezes. When a hedge fund faces redemptions, it is forced to sell its convertible bond holdings, thus driving down prices. Moreover, the risk aversion of convertible bond investors may rise during a market crash and increase the sell-off of convertible bond holdings. A high correlation in the selling activity of hedge funds contributes to the price pressure on convertible bonds during market disruption events. Convertible bonds are one of the asset classes most prone to fire-sale risk. Any exogenous shock in financial markets, even unrelated to convertible bond fundamentals, leads to selling pressure. In the past decade, the convertible bond market experienced periods of serious troughs. For example, convertible bonds reacted negatively to the 1998 crisis after the LTCM collapse. A second example is the 2005 period, when convertible arbitrage hedge funds faced large redemptions from investors after poor past performance. Finally, in October 2008, the post-Lehman credit crunch crisis brought another free fall in convertible bond prices. The risk of forced selling may explain a significant portion of the drawdowns of convertible bond prices during financial crises. Awareness of price pressure and forced-selling risk causes investors to demand a premium in expected returns to hold convertible bonds in their portfolios. Hedge fund price pressure is an additional risk factor in the convertible-bond-pricing model, and this risk is amplified in times of financial turbulence. This is why the underpricing of convertible bonds is more apparent during financial crises.

To conduct the empirical analysis, we use monthly returns of convertible bonds, constituents of the UBS (now Thomson Reuters) Convertible Bond Index, and monthly aggregate flows of convertible arbitrage hedge funds to measure hedge fund demand innovations (*HFD*).[2] First, we estimate the *HFD* beta for equally weighted and value-weighted portfolios of convertible bonds, considering separately the impacts of unexpected flows, inflows, and outflows. The paper finds no significant impact of innovation in inflows on the con-

[2]We use the abbreviation *HFD* to denote hedge fund demand innovations throughout the paper.

temporaneous returns of convertible bond portfolios. However, both unexpected flows and outflows negatively affect contemporaneous convertible bond returns, indicating the presence of price pressure. The value-weighted portfolios have lower estimates of the *HFD* beta than equally weighted portfolios, suggesting that convertible bonds with larger market capitalization are less sensitive to price pressure. Furthermore, the paper documents that the post-Lehman period from September 2008 through March 2009 was characterized by rising selling pressure. We also test for Granger causality between convertible bond returns and hedge fund demand innovations in order to avoid the endogeneity bias in the regressions and find conclusive evidence that convertible bond returns do not cause fluctuations in hedge fund demand innovations.

Second, we construct the price-pressure risk factor by forming an equally weighted long/short portfolio that takes long and short positions in convertible bonds with the highest and lowest *HFD* betas, respectively. The price-pressure risk factor is defined as the out-of-sample next-month return of this long/short portfolio. We rebalance this portfolio each month to obtain the monthly time series for the price-pressure factor. The positive average next-month return spread between the extreme quantile portfolios indicates the existence of a price-pressure risk premium. We then implement the Fama-Macbeth procedure and find a positive, economically significant premium in convertible bond returns that is attributable to price pressure.

Finally, we investigate how the price-pressure factor affects the contemporaneous return of a convertible bond mutual fund. Augmenting the benchmark model for portfolios of convertible bond mutual funds — either CAPM or the multifactor model with equity, bond, and volatility factors — with the price-pressure risk factor, we report that price pressure is a statistically significant factor that increases the explanatory power of the benchmark risk model. The exposure to price pressure has a negative sign, demonstrating that hedge-fund selling pressure diminishes the contemporaneous returns of convertible bond mutual funds.

The structure of the paper is as follows. Section 3.2 discusses the relevant literature on convertible bonds. Section 3.3 describes the data sources for the demand and supply sides of the convertible bond market. Section 3.4 demonstrates the empirical results and discusses their economic significance. Section 3.5 concludes, and tables appear in Appendix 3.6.

3.2 Literature Review

A large body of literature has documented pricing anomalies in convertible bond markets. In many cases, the mispricing is considered relative to a theoretical model. The classic theoretical pricing models of convertible bonds are based on pricing a contingent claim on the firm value. Models of this kind were developed by Ingersoll (1977), Brennan and Schwartz (1977), and Brennan and Schwartz (1980). Following these basic models, various extensions have been developed with the objective of achieving a better empirical fit. However, they were unable to eliminate the mispricing completely. Ammann et al. (2003) use a sample of French convertible bonds to investigate the ability of stock-based binomial-tree models with exogenous credit risk to price a variety of convertible bond specifications. The authors find that market prices are on average 3% lower than the ones implied by their model. These results are consistent with the earlier empirical evaluation of underpricing of 3.75% for a sample of U.S. convertible bonds conducted by King (1986).

There are many potential explanations for convertible bond underpricing. For example, Calamos (2003) attributes the underpricing phenomenon to the underestimation of stock volatility. In contrast, Lhabitant (2002) argues that the mispricing is driven by the complexity of convertible bond valuation. Agarwal et al. (2011) suggest that convertible bonds are mispriced due to illiquidity, small issue size, and the complexity of pricing models. The literature has paid special attention to the short-term and long-term performance of new issues and the mispricing of convertible bonds at issue. The performance of new issues is studied by Henderson (2005), who reports that convertible bonds are underpriced at issuance and consequently have positive forward risk-adjusted returns.

Choi et al. (2009) document an increase in the short-selling of stocks around the period of convertible bond issuance. This observation suggests that the majority of new issues are bought by convertible bond arbitrageurs. Choi et al. (2010) further their analysis of the interrelationship between convertible arbitrage hedge funds and the issuance of convertible bonds, documenting that issuance is positively related to several measures of capital supply by arbitrageurs, namely hedge fund returns, flows, and leverage.

Our paper approaches convertible bond mispricing by detecting the price pressure transmitted by demand fluctuations. Mitchell et al. (2007) investigate how capital-constrained arbitrageurs affect the convertible bond market. The authors compare the cheapness of convertible bonds (the spread between the theoretical price and the market price) and returns of hedge funds that report large convertible bond holdings during two crises: the period of convertible arbitrage hedge fund redemptions from December 2004 to June 2006 and the period of the LTCM failure from December 1997 to December 1999. They also conduct a similar analysis of the activity of merger arbitrageurs and their impact on the prices of merger targets during the 1987 market crash. In both cases, the authors report significant price deviations from theoretical values in situations of capital shocks when hedge funds are unable to provide liquidity to market participants. Our paper represents an extension of the event analysis by Mitchell et al. (2007). Due to the niche position of the convertible bond market and the unique role of convertible arbitrage hedge funds as specialized investors, our paper argues for the existence of not only event-specific but also systematic mispricing driven by hedge fund demand.

Agarwal et al. (2011) empirically study the source of return and risk of convertible arbitrage hedge funds. The authors construct a multifactor model whose factors are portfolios mimicking the convertible arbitrage strategy: positive carry, volatility arbitrage, and credit arbitrage. The abnormal returns (relative to these systematic risk factors) of convertible arbitrage hedge funds are explained by the embedded liquidity premium. Furthermore, they compute the supply-demand imbalance factor and show its statistical significance in explaining the cross-sectional returns of convertible arbitrage hedge funds. The authors find no alpha in convertible arbitrage hedge fund returns after accounting for three asset-based style factors and a liquidity premium. Batta et al. (2010) also support the liquidity-based explanation of convertible arbitrageurs' alpha. The price-pressure factor constructed in our paper is related to the supply-demand imbalance factor used by Agarwal et al. (2011). Both factors provide a liquidity signal in the convertible bond market. However, the main focus of Agarwal et al. (2011) is to examine how the supply of convertible bonds affects the returns of convertible arbitrageurs. On the contrary, our paper contributes to understanding how hedge fund demand influences the pricing of convertible bonds. We consider hedge fund price pressure as a factor explaining the mispricing puzzle of convertible bonds.

The portfolio holdings of a convertible bond mutual fund are mainly long-only positions in convertible bonds. The only study of convertible bond mutual funds was conducted by Ammann et al. (2010). According to their estimates, assets under management of convertible bond mutual funds that report to the CRSP mutual fund database were equal to USD 40 bn in 2005. The authors examine empirically a large set of systematic factors that affect returns of convertible bond mutual funds. The multifactor model for convertible bond mutual funds includes

- stock factors: *EMKT*, *SMB*, *HML*, and *MOM* as in the four-factor model proposed by Carhart (1997);
- bond factors: the term factor *TERM* is defined as the return of the Lehman (now Barclays Capital) US Government Long Bond Index minus the 1-month Treasury bill rate; the default factor *DEF* is defined as the return on the Lehman US Corporate Long Bond Index minus the return of the Lehman US Government Long Bond Index; the high yield factor *HY* is defined as the return on the Merrill Lynch US High Yield

Index; and the broad bond factor $BOND$ is defined as the excess return of the Lehman US aggregated Government/Credit Bond Index;

- option factors: $VOLA$ is defined as the return on the CBOE (Chicago Board Options Exchange) Volatility VXO Index; $\max(0, MKT - BOND)$; MKT^2 is the squared market factor; ATM is defined as the return on a dynamic portfolio of at-the-money call and put options; and OTM is defined as the return on a dynamic portfolio of out-of-the-money call and put options; and

- convertible-bond-specific factors: CBI is defined as the return on the Merrill Lynch All US Convertible Bond Index; $CBAI$ is defined as the return on the CSFB/Tremont (now Credit Suisse/Tremont) Convertible Arbitrage Index; and SD is the supply-demand factor proposed by Agarwal et al. (2011).

A parsimonious version of this multifactor model with the equity market factor, two bond factors, and one volatility factor is used as a benchmark model in our paper.

The existence of price pressure is not unique to the convertible bond market. For example, Coval and Stafford (2007) document the existence of an equity market price-pressure effect created by institutional investors. They find empirical support for the hypothesis that capital flows forcing the selling or buying activity of mutual funds create price pressure on the equity market that leads to the persistent mispricing of stocks relative to their fundamental values. Since mutual fund holdings information is publicly available, the authors can measure the flow-motivated trading in a given stock and define aggregate price pressure. In our case, we cannot follow this procedure, since hedge fund holdings are unavailable; therefore we must rely on estimated sensitivities of individual convertible bonds to demand shocks to construct the price-pressure factor.

As hedge funds are liquidity providers in the convertible bond market, research on aggregate liquidity and its impact on asset prices is also relevant for this paper. A number of papers show that aggregate market liquidity is a persistent and priced risk factor for any asset class. First, Acharya and Pedersen (2005) derive an equilibrium asset-pricing model with a liquidity factor and discover the channel through which liquidity affects asset prices. According to their model, negative liquidity shocks lead to low contemporaneous asset returns and high long-term asset returns. This fact predicts that during downturns, investors are unlikely to sell high-liquidity beta assets, and instead sell low-liquidity beta assets. Second, Pastor and Stambaugh (2003) construct a liquidity factor based on the sensitivity of returns to trading volume and show that expected stock returns are related to the liquidity factor in the cross section. The expected rate of return of stocks with high sensitivity to liquidity is found to be higher than the expected returns of stocks with low sensitivity. Moreover, the authors find that part of the returns of momentum strategies can be attributed to the liquidity factor. Third, Amihud (2002) takes the average daily ratio of absolute stock return to dollar volume as an illiquidity measure and demonstrates that there is a positive relationship between illiquidity and cross-sectional stock returns. Furthermore, the illiquidity premium may be useful in tackling the equity premium puzzle. Finally, Acharya et al. (2013) compute the illiquidity premium for the corporate bond market and additionally discover that illiquid bonds have higher yields.

3.3 Convertible Bond Market: Data and Definitions

This section describes our data sources and the participants in the convertible bond market; it also defines the variables of interest for the empirical analysis. The supply side is represented by the convertible bond issues of U.S. firms, while the demand side is represented by the universe of convertible arbitrage hedge funds. Furthermore, we employ a sample of convertible bond mutual funds in our study.

3.3.1 Convertible Bond Supply

Our sample of convertible bonds consists of constituents of the UBS Global Convertible Index, a broad-based index representing the convertible bond market that is often used as a benchmark for convertible bond mutual funds. It is defined as a market capitalization-weighted total return index; that is, coupons are assumed to be reinvested upon payment proportionate to index weighting. The index was launched on September 30, 1998; however, historical data for all its constituents are available only since September 30, 2002. The constituents of the index are any equity-linked convertible instruments such as convertible bonds, exchangeables, mandatory issues, bonds with warrants, and similar products that must be convertible into a listed share. For example, pre-IPO convertibles are excluded from the index. The issues are required to satisfy liquidity criteria (a valid indicative price is available from three or more market makers) and market capitalization criteria (the size of the issue is above USD 100M). The index was devised by UBS and is independently maintained by MACE Advisers. About 50% of global issues originate in the United States, and our paper reports results for the sample of U.S. convertible bonds.[3] The data sample covers the period from October 2002 to September 2009.

The return of a convertible bond is computed as a price differential and an interest accrual using the following formula:

$$R_t = \frac{P_t + I_t}{P_{t-1}} - 1, \tag{3.1}$$

where P_t is the convertible bond price at the end of month t and I_t the interest payment accumulated in the period from $t-1$ to t. The returns are collected at monthly frequency. Panel A of Table 3.1 reports the cross-sectional mean, median, and standard deviation of the distributional properties of convertible bond returns. Convertible bonds have average

[3] Among other regions, Japanese convertible bonds are the most prominent in the Asian market, and French issues dominate the European market.

annualized returns of 10% and an average volatility of 22% or, equivalently, a Sharpe ratio of 0.45. Median values of returns are lower: 6.6% with approximately the same volatility, yielding a Sharpe ratio of 0.3. Based on the median value of the Jarque-Bera test statistics, we are unable to reject the hypothesis that convertible bond returns are normally distributed.

Table 3.2 reports convertible bonds' characteristics for each year. The average number of securities in the convertible bond universe decreased to 189 in 2009 from 292 in 2002. Average market capitalization of convertible bond issues varies from USD 583M in 2002 to USD 798M in 2008. However, the total market capitalization of convertible bonds in the data sample achieves its maximum value USD 210 bn in 2007, after which it decreases to USD 135 bn in 2009. Monthly average returns of convertible bonds were negative in 2008, with a subsequent recovery in 2009. Convertible bond returns were also low in 2005 and 2007. The credit crunch crisis in 2008 had a significant impact on the convertible bond market, with negative returns and an increase in volatility. Average delta, a measure of equity sensitivity, had its peak value of 62% in 2007 and decreased after the crisis to 43% in 2009.

3.3.2 Conventional Multifactor Convertible Bond Model

We conjecture that the returns of convertible bonds are driven by conventional risk factors such as the equity market factor, two bond factors, and the volatility factor. This conjecture is based on the arbitrage pricing theory (APT) multifactor model for convertible bond performance presented by Ammann et al. (2010). This set of risk factors reflects the hybrid structure of convertible securities with risk exposures coming from the corporate bond, the underlying equity, and the embedded option to convert the bond into equity. The conventional factors are constructed for the U.S. market. The equity factor is the equity market return in excess of the risk-free rate where the market includes all NYSE, AMEX, and NASDAQ stocks. The risk-free rate is the 1-month T-bill rate of return. The data come from the K. French data library.[4] There are two bond factors: the term spread and the default spread. The data are collected from the FRED database of the Federal Reserve Bank of St. Louis. We take Moody's Seasoned Aaa and Baa Corporate Bond Yields plus the 1-Year $GS1$ and 10-year $GS10$ Treasury Constant Maturity rates. The credit risk or default spread is measured by the difference between Baa and Aaa yields. The term spread is defined as the difference between $GS10$ and $GS1$, and captures variation in the slope of the yield curve. Finally, the volatility factor is represented by the monthly percent change in the CBOE Volatility Index (VIX index). The month-end closing prices for the VIX index are taken directly from the CBOE website.[5]

Table 3.3 reports the estimated coefficients of the convertible bond multifactor model with conventional risk factors for the monthly returns of equally weighted and value-weighted portfolios. In additional model specifications, we augment the regression by the dummy variable for the official recession period declared by the the National Bureau of Economic Research (NBER) from January 2007 to June 2009. The general form of the multifactor model is the following:

$$R_{CB,t} = \alpha + \sum_{i=1}^{M} \beta_i F_{i,t} + \beta_r REC_t + \varepsilon_t, \tag{3.2}$$

where $R_{CB,t}$ is convertible bond portfolio return in excess of the risk-free rate, α is an intercept coefficient, β_i are factor loadings, REC_t is the NBER recession dummy variable, ε_t is the regression residual, and $F_{i,t}$ are factor returns such that

$$\sum_{i=1}^{M} \beta_i F_{i,t} = \beta_1 EMKT_t + \beta_2 TERM_t + \beta_3 DEF_t + \beta_4 VIX_t, \tag{3.3}$$

[4]The risk-free rate comes originally from Ibbotson Associates. The K. French data library is available at: `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

[5]Data are publicly available from `www.cboe.com`.

where $EMKT_t$ is the excess equity market return, $TERM_t$ is the term spread, DEF_t is the default spread, and VIX_t is the monthly change in the VIX index.

The most significant factor is the equity market factor, with estimated market betas of 0.63 and 0.58 for the equally weighted and value-weighted portfolios, respectively. This result is consistent with our intuition that equity risk is a major source of risk for a convertible bond investor. The default spread is another significant factor for convertible bonds because credit risk is a natural source of risk for a fixed-income security. The default spread beta is estimated to be equal to 0.68 (0.77) for the equally weighted (value-weighted) portfolio. The estimated coefficients of the default spread dramatically increase in the model specification with the recession dummy to 1.47 (1.59) for the equally weighted (value-weighted) portfolio. Therefore, the sensitivity of convertible bond returns to a widening credit spread increases during the recession. The recession negatively affects convertible bond returns, as indicated by the statistically significant negative coefficient on the recession dummy variable. Contemporaneous returns of individual bonds are negative during the recession due to the deterioration of macroeconomic conditions and the decrease in equity prices. Both high-delta convertible bonds and high-yield junk bonds are negatively affected: the former due to the fall in equity prices and the latter due to the deterioration of convertible bond creditworthiness. Comparing the coefficients for the value-weighted and equally weighted portfolios, it is worth noting that the equity market beta is lower and the credit spread beta is higher for convertible bonds with a larger market capitalization. Interest-rate risk reflected in the government bond spread is insignificantly negative. The volatility factor is also found to be insignificantly negative in this linear regression framework. Nonetheless, we use all conventional factors to control for the systematic risk premium in the regression analysis in this paper.

3.3.3 Hedge Fund Demand for Convertible Bonds

We act under the assumption that the demand for convertible bonds is driven by convertible arbitrage hedge funds. This assumption is based on the facts reported by Mitchell et al. (2007), Choi et al. (2009), and Choi et al. (2010). However, the assumption of homogenous demand can be considered as a limitation of the empirical analysis since the magnitude of the price-pressure impact depends on the validity of this assumption. Nonetheless, we argue that the major source of convertible bond capital is provided by hedge funds, at least in the first decade of the twenty-first century, in which our sample period from October 2002 to September 2009 is included.

The hedge fund data source is the TASS Lipper Hedge Fund Database. The data include information about monthly returns, assets under management, and different hedge fund characteristics. The data contain both live and graveyard hedge funds. Hedge funds are divided into ten primary investment categories, one of which is convertible arbitrage. The CSFB/Tremont Hedge Fund Style indices aggregate the returns and assets under management (AUM) of hedge funds for each investment strategy. The style index aims to be a representation of the hedge fund strategy universe and covers approximately 85% of the AUM in each investment category, according to the CSFB description.[6]

We use the CSFB/Tremont Convertible Arbitrage Hedge Fund Index in order to measure hedge fund demand. An alternative method would be to use the individual hedge fund flows and compute the aggregate measure. However, we choose to use the index data to ensure representativeness of convertible arbitrage hedge funds. The demand is measured by hedge fund flows, calculated from return and assets under management by the following formula:

$$Flows_{t+1} = \frac{AUM_{t+1} - (1 + R_t)AUM_t}{AUM_t}, \tag{3.4}$$

where AUM_t is assets under management at month t and R_t is the hedge fund return during

[6]Descriptions of the CSFB/Tremont Hedge Fund indices are publicly available at *www.hedgeindex.com*.

month t (observable at the beginning of month $t+1$). This definition of flows is widely accepted and used in the literature.[7] Basic descriptive statistics of flows are presented in panel C of Table 3.1.

Following Baquero and Verbeek (2009), we assume that flows are predictable and persistent over time. Hence, we can separate expected and unexpected flows. We apply the simple autoregressive prediction process $AR(p)$ for flows and estimate the residuals from the following time-series regression:

$$Flows_t = \gamma_0 + \sum_{i=1}^{p} \gamma_i Flows_{t-i} + \widehat{\varepsilon}_t. \tag{3.5}$$

The number of lags is chosen according to the Akaike's information criterion (AIC)[8] and is equal to four.[9] The regression residuals represent hedge fund demand innovation,[10] which is the unexpected portion of the total hedge fund demand for convertible bonds.

The regression output appears in panel A of Table 3.4. The adjusted R^2 coefficient of the $AR(4)$ regression is 0.44, and all individual lag coefficients are statistically significant. Panel B of Table 3.4 reports the pairwise Pearson, Kendall, and Spearman correlation coefficients between hedge fund demand surprise and conventional convertible bond risk factors. We find a positive Pearson correlation of hedge fund demand with the equity market factor of 0.24 and a negative correlation with the default spread of -0.28. On the one hand, a widening credit spread during recession negatively affects the buying opportunities for convertible bonds. The demand for convertible bonds shrinks due to the increase in credit risk. On the other hand, the demand for convertible bonds increases

[7] For example, see Fung et al. (2008).

[8] We chose the AIC as the most widely known and used model selection criterion.

[9] A conventional way to preprocess fund flows in the literature is to apply moving average smoothing. Fung et al. (2008) and Ozik and Sadka (2010) use the arithmetic sum of the prior three months' flows. Our $AR(4)$ model essentially provides a similar preprocessing of the flows. The residuals from the $AR(4)$ process applied to the raw flows and the $AR(1)$ process applied to $MA(3)$-smoothed flows have a correlation of 0.8.

[10] The terms hedge fund demand innovation, hedge fund demand shock, hedge fund demand surprise, and unexpected hedge fund demand are used interchangeably throughout the paper.

during a bull equity market, which is indicated by a positive correlation with the equity risk factor. However, the significance of correlations diminishes in the case of alternative types of correlation that are more robust to outliers. Hence it is likely that the higher correlation may be attributed to certain time periods (of equity market expansion and recession with corresponding credit spread widening) within the sample history.

3.3.4 Convertible Bond Mutual Funds

A typical long-only convertible bond portfolio can be represented by the holdings of a convertible bond mutual fund. As a long-only specialized investor in convertible bonds, this category of mutual funds is expected to be sensitive to the price-pressure risk factor generated by convertible arbitrage hedge funds. The data for convertible bond mutual funds (defined by the Morningstar category as "convertible") are taken from the Yahoo Finance Database. Our sample consists of 40 mutual funds with sufficient historical data. Monthly returns are computed as price differentials adjusted for dividends payments.[11] Descriptive statistics of convertible bond mutual funds are reported in panel B of Table 3.1. Average returns of convertible bond mutual funds are 4.8% with a volatility of 14.5%, and the median returns are 6.3% with a volatility of 12.8%. The average Sharpe ratio is 0.22, and its median is 0.32. Comparing these numbers with convertible bond returns, we observe that the median risk-adjusted returns in terms of Sharpe ratio are the same while the average risk-adjusted returns are lower for convertible mutual funds. Moreover, contrary to convertible bond returns, convertible bond mutual fund returns are unlikely to follow the normal distribution because we reject the Jarque-Bera Gaussian distribution hypothesis based on both average and median values of the Jarque-Bera test statistic.

[11] The typical mandate of convertible bond mutual funds allows minor holdings in other securities including stocks or bonds converted to stocks, and stocks' dividend payout is a part of the return calculation.

3.4 Estimating the Price-Pressure Factor

In this section, we examine empirically the impact of hedge fund demand shocks on contemporaneous convertible bond returns and construct the price-pressure factor. Furthermore, we estimate the risk premium for bearing price-pressure risk in convertible bonds. Finally, we investigate how the price-pressure risk factor affects the monthly returns of convertible bond mutual funds.

3.4.1 Does Hedge Fund Demand Matter?

Before constructing the price-pressure risk factor, we pose the simple questions of whether a hedge fund impact exists and, if so, is statistically significant in the time series of convertible bond returns. We address these questions by regressing the contemporaneous returns of the equally weighted and value-weighted convertible bond portfolios on hedge fund demand surprise, controlling for the conventional convertible bond factors. We consider innovations in flows, inflows (positive net flows), and outflows (negative net flows) separately in different regression specifications.[12] We expect an asymmetry in the magnitude of the impact of outflow shocks versus inflow shocks, as selling pressure may have a more prominent effect than buying pressure on convertible bond prices.

The time-series regression model is the following:

$$R_{CB,t} = \alpha + \sum_{i=1}^{M} \beta_i F_{i,t} + \beta_Q Q_t + \varepsilon_t, \tag{3.6}$$

where $R_{CB,t}$ is the monthly return of the convertible bond portfolio in excess of the risk-free rate; Q_t is the hedge fund demand surprise measured by innovation in flows, inflows, or outflows; $F_{i,t}$ are the conventional convertible bond factors; β_i are factor loadings; and α

[12]Positive or negative net flows are not necessarily inflows or outflows; however, we use this terminology to distinguish the signs of flows.

is an intercept coefficient.

Table 3.5 reports the regression output. We observe the negative coefficient on the flows' surprise, which is equal to -0.15 (-0.11) with a t-statistic of -1.57 (-0.11) for the equally weighted (value-weighted) portfolio. This result may be considered counterintuitive. However, the impact of inflows' surprise is insignificant while the coefficient on the outflows' surprise is highly significant and equals -0.26 (-0.20) with a t-statistic of -2.41 (-1.93) for the equally weighted (value-weighted) portfolio. The impact of unexpected flows on contemporaneous convertible bond returns is driven predominately by innovation in outflows of capital from convertible arbitrage hedge funds rather than innovation in inflows. This impact of outflow shocks is so strong that the coefficient on overall flows' surprise remains negative even though its statistical significance is weaker compared with outflows. The impact of surprise in both flows and outflows is weaker for the value-weighted portfolios, indicating that securities with lower market capitalization and lower ex-ante liquidity are more sensitive to outflow shocks.

To avoid a potential endogeneity bias in the return and flows relationship, we conduct the Granger causality test for the equally weighted and value-weighted portfolios of convertible bonds versus innovation in flows, inflows, and outflows. The results are reported in Table 3.6 for six model specifications. The test is conducted for 1 lag and at the 5% level of significance. We cannot reject the null hypothesis that convertible bond returns do not cause a surprise in flows for any of these model specifications. However, we reject the null hypothesis that surprise in flows does not cause convertible bond returns for the value-weighted portfolios. The results are weaker in the case of equally weighted portfolios, but the values of the F-statistics are close to the critical value. To sum up, it is unlikely that convertible bond returns cause flow shocks; however, the opposite direction of causality is plausible.

The preliminary regression analysis motivates us to use the capital outflow surprise in modeling the selling pressure generated by convertible arbitrage hedge funds. On the one hand, selling pressure may lead to fire sales of convertible bonds, and thus the outflows measure helps to identify fire-sale risk. On the other hand, the persistence of selling pressure explains the perceived underpricing puzzle of convertible bonds. We let *HFD* denote the surprise in outflows from convertible arbitrage hedge funds.

3.4.2 Price-Pressure Factor

As shown in the previous section, the hedge fund demand innovation *HFD* has a significant negative impact on the contemporaneous returns of convertible bond portfolios, and this impact is stronger for smaller-cap securities. To measure the price pressure in the convertible bond market, we sort convertible bonds by their *HFD* betas and construct the factor as a factor-mimicking portfolio.

The *HFD* betas for individual convertible bonds are estimated using OLS for a sample window of 24 months while controlling for the conventional convertible bond factors. Control variables allow us to focus on the incremental impact of the hedge fund demand surprise. The regression equation has the following form:

$$R_{CB,t} = \alpha + \sum_{i=1}^{M} \beta_i F_{i,t} + \beta_{HFD} HFD_t + \varepsilon_t, \tag{3.7}$$

where $R_{CB,t}$ is the monthly convertible bond return in excess of the risk-free rate at time t, HFD_t is the innovation in outflows from convertible arbitrage hedge funds, α is an intercept coefficient, β_i and β_{HFD} are the factor loadings, and $F_{i,t}$ are conventional convertible bond factors: excess equity market return, term spread, default spread, and changes in the VIX index.

We rank convertible bonds by the value of the t-statistic for the *HFD* beta estimates. Ranking by t-statistic rather than by beta values is a more robust approach to the presence of outliers that cause large but statistically insignificant coefficients. Convertible bonds are assigned to three quantile portfolios (*Q1*, *Q2*, and *Q3*) based on the estimated *HFD* beta with increasing beta t-statistic from the bottom quantile to the top quantile. We chose three quantiles to have a sufficient number of securities in each portfolio. Thus the factor portfolio is more diversified and less driven by idiosyncratic sources of risk and return. The quantile portfolios are equally weighted.[13] We calculate the post-ranking out-of-sample monthly returns of the quantile portfolios and the spread return between the top and the bottom quantile portfolios. The portfolios are rebalanced each month, and the size of the estimation window for the *HFD* coefficient remains fixed. The spread represents the return on a zero-investment portfolio that takes equally weighted long and short positions in the top and bottom quantiles, respectively. This factor-mimicking portfolio of convertible bonds defines the price-pressure risk factor.

Panel A of Table 3.7 shows descriptive statistics of the *HFD* beta-sorted portfolio returns: mean, values of t-statistic and of Newey-West t-statistic adjusted for autocorrelation and heteroscedasticity, median, maximum, and minimum values, standard deviation, and skewness and kurtosis coefficients. We observe a monotonically increasing pattern from the bottom to the top quantile portfolio in the time series of mean values, median values, and corresponding t-statistic. This pattern allows us to identify the positive spread between the top and bottom quantile portfolios that implies the existence of a premium for holding convertible bonds with higher *HFD* beta. An investor holding convertible bonds requires higher average returns for holding securities with high *HFD* beta. The premium is reflected in the positive return of the spread portfolio. The average value of the price-pressure factor return equals 0.32 with a t-statistic of 1.39 (1.07 for the Newey-West t-statistic), which is higher than corresponding values of t-statistics for the quantile portfolios. We consider these values of the t-statistic adequate to identify the price-pressure effect given the relatively short time period of the sample. However, it is worth examining the t-statistic relative to alternative systematic risk factors that we present in Table 3.8.

Panel B of Table 3.7 reports the characteristics of the constituents of the *HFD* beta-

[13]The results for value-weighted quantile portfolios are presented in the robustness check section.

sorted portfolios. First, we estimate the post-ranking *HFD* betas of the quantile portfolios. The *HFD* beta increases from -1.14 in the bottom quantile to 1.20 in the top quantile and is highly significant in both extreme quantiles. This observation is consistent with the ex-ante choice of convertible bonds in the quantile portfolios. Delta, the measure of equity sensitivity, is slightly decreasing. The bottom quantile portfolio is characterized by a high interest-rate spread.[14] We find a hump-shaped pattern in market capitalization. Small-cap convertible bonds usually have significant *HFD* betas while large-cap securities are not sensitive to hedge fund demand shocks. This phenomenon may be also linked to liquidity, as convertible bonds with large market capitalization are ex ante more liquid securities.[15] Therefore price-pressure risk is more relevant for small-cap convertible bond issues.

Table 3.8 reports the time-series estimates of factor exposures for the *HFD* beta-sorted portfolios and the price-pressure-factor-mimicking portfolio. Absence of significant exposures to convertible bond factors would indicate the existence of a risk premium orthogonal to these alternative sources of risk. Panel A of Table 3.8 reports the regression coefficients of the four-factor model with conventional convertible bond factors: excess equity market return, term spread, default spread, and changes in the VIX index. Panel B of Table 3.8 reports regression coefficients for the one-factor CAPM setting with only the equity market factor. All factor exposures except the equity market exposures are insignificant for the price-pressure factor. In the case of the four-factor regression, the equity market factor is statistically significant for the quantile portfolios, but the statistical significance (t-statistic of -1.72) drops for the price-pressure factor. The estimates of the intercept coefficient for the price-pressure factor are positive and statistically significant: 1.02 with a t-statistic of 2.35 for the four-factor regression and 0.72 with a t-statistic of 2.3 for the one-factor

[14]The interest-rate spread is defined as the difference between the coupon rate of the convertible bond and the risk-free rate. It captures both term spread and default spread.

[15]The relationship between the price-pressure factor and the market-wide liquidity factor is a subject of future research.

regression. Therefore we conclude that the premium attributed to price pressure cannot be fully captured by the equity market premium or alternative factors.

3.4.3 Price-Pressure Premium Estimation

In this section we estimate the premium attributed to the price-pressure factor using the two-stage regression procedure pioneered by Fama and MacBeth (1973).[16] The objective of the Fama-Macbeth approach is to quantitatively estimate the magnitude of the risk premium associated with a particular factor and the pricing error of the multifactor model. The two-pass procedure combines estimating the factor exposures from time-series regressions with estimating the risk premia from cross-sectional regressions. Applying this procedure, we first investigate whether adding the price-pressure risk factor brings an added value to the pricing of convertible bonds and decreases the pricing error of a multifactor model; second, we measure the premium attributable to the price-pressure factor relative to conventional factors.

First, we create a set of test portfolios of convertible bonds, for which we run the Fama-MacBeth regressions. We avoid testing on individual securities because estimates of factor exposures for portfolios are more likely to be statistically significant and stable over time. We apply a simultaneous 3×3 double-sorting procedure such that convertible bonds are sorted on the one hand by the *HFD* beta and on the other hand by their market capitalization. We use 24-month rolling time-series regressions to estimate the *HFD* beta. The sample length of 24 months remains fixed. The returns of test portfolios are computed outside of the estimation window. This formation procedure for the test portfolios allows us to smooth the individual characteristics of convertible bonds and to avoid the formation of similar portfolios with index-like behavior that might occur if the sampling were done randomly. As a robustness check, we also create a similar sorting procedure based on

[16] Refer to Cochrane (2005) for full details of asset pricing tests including the Fama-MacBeth approach or to the recent survey of empirical cross-sectional asset pricing by Goyal (2012).

convertible bond delta.

There are two stages in the Fama-Macbeth procedure. In the first stage, we obtain the estimates of factor exposures from N time-series regressions, where N is the number of test portfolios. In each regression, the contemporaneous returns of the convertible bond portfolios in excess of the risk-free rate are regressed on the returns of K risk factors:

$$R_t^n = a_n + \sum_{k=1}^{K} \beta_{n,k} F_{t,k} + \varepsilon_t^n, \text{ for each } n \in \{1, ..., N\}, \tag{3.8}$$

where R_t^n is the excess return of the n^{th} test portfolio at time t, a_n is the time-series intercept, $\beta_{n,k}$ is a factor exposure for the n^{th} test portfolio and the k^{th} factor, $F_{t,k}$ is the k^{th} factor return at time t, and ε_t^n is the regression residual for the n^{th} regression.

In the second stage, we conduct T cross-sectional regressions using the matrix $\beta_{n,k}$ of estimated factor exposures as a regressor matrix in each regression. The regression specification is as follows:

$$R_n^t = \lambda_0^t + \sum_{k=1}^{K} \beta_{n,k} \lambda_k^t + \alpha^t, \text{ for each } t \in \{1, ..., T\}, \tag{3.9}$$

where T is the size of the sample (excluding the first 24 months, when returns of the test portfolios sorted by *HFD* beta are unavailable), R_n^t and $\beta_{n,k}$ are as in equation (3.8), λ_k^t is the estimated risk premium at time t for the k^{th} factor, λ_0^t is an intercept term at time t to denote the zero-beta rate in excess of the risk-free rate, and α^t is a residual that denotes the pricing error at time t. The cross-sectional regressions are run using OLS. The factor risk premia $\widehat{\lambda}_k$ and the pricing errors of the model $\widehat{\alpha}$ are computed as time-series averages:

$$\widehat{\lambda}_k = \frac{1}{T} \sum_{t=1}^{T} \widehat{\lambda}_k^t \text{ and } \widehat{\alpha} = \frac{1}{T} \sum_{t=1}^{T} \widehat{\alpha}^t. \tag{3.10}$$

The variances of the estimates $\widehat{\lambda}_k$ and $\widehat{\alpha}$ are computed as the variances of the average estimates as follows:

$$var(\widehat{\lambda}_k) = \frac{1}{T^2}\sum_{t=1}^{T}(\widehat{\lambda}_k^t - \widehat{\lambda}_k)(\widehat{\lambda}_k^t - \widehat{\lambda}_k)' \text{ and } var(\widehat{\alpha}) = \frac{1}{T^2}\sum_{t=1}^{T}(\widehat{\alpha}^t - \widehat{\alpha})(\widehat{\alpha}^t - \widehat{\alpha})'. \quad (3.11)$$

To account for possible autocorrelation in risk premium estimates, we also adjust the statistics by the Newey-West corrections.

The results of the second-stage estimation are displayed in Table 3.9. We run regressions for two model specifications: the one-factor CAPM with the equity market factor (model I) and the four-factor convertible bond model (model III). Both models are augmented by adding the price-pressure factor (models II and IV). We find negative equity premia in convertible bond returns and a positive default premium. The price-pressure premium is positive and equal to 0.38 regardless of the multifactor benchmark model. Both the intercept term and the equity premia are highly statistically significant for all model specifications except model IV. In the case of model IV (the four-factor model augmented by the price-pressure factor), the statistical significance of the intercept and equity premium decreases. Generally, the statistical significance of the price-pressure and default premia are lower than the equity market premium; however, the relative magnitude of the t-statistic is comparable between these two premia. Furthermore, we observe that the addition of the price-pressure factor to the multifactor model improves the average explanatory power of the second-stage regressions from 0.07 to 0.44 in models I and II and from 0.07 to 0.16 in models III and IV. Table 3.9 provides evidence that, first, the premium attributed to price pressure does not depend on conventional convertible bond factor premia, and second, that it is not negligible relative to alternative factors even in the small sample of given test portfolios.

3.4.4 Convertible Bond Mutual Funds and Price-Pressure Factor

If price pressure is a priced risk factor for convertible bonds, it should appear as an attribute of risk modeling for any convertible bond investor. We test the hypothesis of a missing risk factor for U.S. convertible bond mutual funds. Table 3.10 presents OLS regressions of the equally weighted portfolio returns of convertible bond mutual funds on the price-pressure risk factor. We consider the ability of different benchmark models to explain the returns of convertible bond mutual funds. Panel A shows the one-factor CAPM with the equity market factor and the four-factor convertible bond model with equity market factor, term spread, default spread, and changes in the VIX index. Panel B shows the standard equity factor models: the Fama-French factor model with equity market factor, value factor *HML* (high minus low), and size factor *SMB* (small minus big), and the Carhart model with the additional equity momentum factor *MOM*. All these benchmark models are augmented by adding the previously constructed price-pressure factor *PP* and the price-pressure factor conditional on the post-Lehman period $PP \cdot PL$: the dummy variable for the post-Lehman period from September 2008 to March 2009 multiplied by the price-pressure factor. By adding the latter variable, we identify the selling pressure on convertible bonds, particularly during the 2008 financial crisis.

Adding the price-pressure factor significantly improves, in fact almost doubles, the explanatory power of all regression models: from 0.12 to 0.26 in the case of the four-factor convertible bond model in panel A and from 0.17 to 0.29 in the case of the Carhart model in panel B. The price pressure coefficient is highly statistically significant and around -0.9 for all models. The post-Lehman period is the most prominent example of selling pressure in convertible bonds. We find that the coefficient on the conditional price pressure is significant, and the value is estimated to be in the range of -1.7 to -2.3. Moreover, the adjusted R^2 coefficient increases to 0.35.

In conclusion, we find that traditional equity, bond, and volatility factors poorly explain the performance of convertible bond mutual funds due to the peculiarities of the convertible bond market. By contrast, the proposed price-pressure risk factor captures a significant portion of convertible bond returns and can improve on the existing benchmark risk model. The negative sign on mutual funds' exposure toward the price-pressure factor indicates hedge fund selling pressure. The contemporaneous returns of convertible bond mutual funds as representative long-only portfolios are affected by the underpricing phenomenon of convertible bonds caused by hedge fund selling pressure.

3.4.5 Robustness Checks

In this section, we investigate the robustness of our results. We first consider an alternative portfolio formation method to construct the price-pressure factor using value-weighted *HFD* beta-sorted quantile portfolios. Table 3.11 describes the statistics of the quantile portfolios and the price-pressure factor return. The mean return of the spread portfolio is higher than for the equally weighted portfolio and equals 0.71 with a t-statistic of 2.07. We confirm the positive significant spread and the increasing pattern in quantile portfolio returns from the bottom quantile to the top quantile. The post-ranking *HFD* betas in the top and bottom value-weighted quantile portfolios are higher in absolute value than for the equally weighted quantile portfolios. This finding confirms the observation that larger market-cap convertible bonds are less sensitive to hedge fund demand shocks.

Table 3.12 reports the exposures of the quantile portfolios and the price-pressure factor to alternative risk factors: four conventional bond factors in panel A and a univariate equity market factor in panel B. The only statistically significant exposure for the price-pressure factor is the equity market exposure, which can be explained by the fact that large market-cap convertible bonds are contained in the extreme quantile portfolios and simple value-weighted positioning does not completely eliminate the equity market exposure. The intercept coefficient of the price-pressure factor is highly significant and equals 1.32. It is slightly higher than the corresponding value for the price-pressure factor based on the equally weighted quantile portfolio.

Table 3.13 reports the regression analysis for convertible bond mutual funds. As in Table 3.10, we regress the portfolio returns of convertible bond mutual funds on the price-pressure risk factor while controlling for the benchmark factors. Panel A of Table 3.13 reports regression coefficients using the equity market factor and the conventional convertible bond factors as benchmark models. Panel B of Table 3.13 reports regression coefficients using the Fama-French factors and the momentum factor as benchmark models. The overall fit of the linear model for convertible bond mutual funds is improved by augmenting the factor model with the price-pressure risk factor. We also observe the negative exposure to selling pressure, but it is weaker than in the case of the price-pressure factor based on the equally weighted quantile portfolios.

In addition to using a different portfolio formation rule in constructing the price-pressure factor, we also consider the impact of changing the length of the estimation window to 36 months.[17] The results are consistent with those presented in Section 3.4, indicating the economic significance of the findings throughout the paper. However, the statistical significance is on average slightly lower. The weak statistical significance is a natural consequence of the relatively short time period of the data sample available for the empirical analysis.

Our final robustness check is to investigate the impact of changes in the formation of the test portfolios for the Fama-Macbeth procedure. Table 3.14 documents the results of estimating the price-pressure premium for the portfolios double-sorted by *HFD* beta and delta. The results are very similar to those in Table 3.9. The price-pressure premium is estimated to be 0.34 for the four-factor model with conventional convertible bond factors and 0.46 for the one-factor CAPM setting.

[17] The results are available upon request.

3.5 Conclusion

This paper sheds light on the impact of demand shocks on the risk and return of convertible bonds. The convertible bond market is characterized by a relatively homogeneous demand that is primarily driven by convertible arbitrage hedge funds. Arbitrageurs in this niche market create persistent selling pressure on convertible bond prices. Our paper empirically tests the hedge fund price-pressure hypothesis as an explanation for the puzzle of convertible bond underpricing.

Demand shocks are measured by innovations in aggregate outflows of convertible arbitrage hedge funds. Using the estimated sensitivities of convertible bond returns to hedge fund demand innovations, we construct a new convertible bond risk factor associated with hedge fund price pressure. The time-series average return of the price-pressure factor is positive, indicating the existence of a positive premium for holding securities whose prices are sensitive to demand shocks. Further, we apply the Fama-Macbeth procedure to estimate the price-pressure factor premium for the set of convertible bond portfolios and discover that it is positive and non-negligible relative to conventional risk factors. Finally, we show that convertible bond mutual funds have significant exposures to price-pressure risk. The existence of hedge fund selling pressure adversely affects contemporaneous returns of convertible bond mutual funds.

This paper ought to be of use to long-only convertible bond investors, who should be aware of the existence of hedge fund price pressure. Adding price-pressure risk provides value in spanning the space of convertible bond risk factors. The robustness of the results is supported by variations in the definition of the price-pressure factor, such as using the value-weighted method of factor-mimicking portfolio formation instead of the equally weighted method, alternative estimation windows for the hedge fund demand beta, and market- and cap-sorted convertible bond test portfolios versus delta-sorted test portfolios in the Fama-Macbeth estimation of the price-pressure premium. Thus, the paper establishes the economic significance of the impact of price pressure on convertible bonds. However, its statistical significance could be improved by analyzing a longer-time, higher-frequency sample of convertible bonds and convertible bond mutual funds, a subject for future research.

3.6 Appendix

Table 3.1: Descriptive statistics for monthly returns of convertible bonds (panel A), monthly returns of convertible bond mutual funds (panel B), and monthly convertible arbitrage hedge fund flows (panel C). Panel A and panel B report cross-sectional mean, median, and standard deviation of the annualized returns, standard deviation, Sharpe ratio, skewness, kurtosis, and Jarque-Bera statistic ($JB = 0$ if the Null that the distribution is Gaussian cannot be rejected vs. $JB = 1$ if the Null is rejected at the 5% significance level). Panel C reports time-series mean, standard deviation, skewness, kurtosis, and Jarque-Bera statistic of convertible arbitrage hedge fund flows. The sample period is from October 2002 to August 2009.

Panel A	**Mean**	**Median**	**Std Dev**
Return	10.58	6.59	22.54
Std Dev	22.31	18.30	17.10
Sharpe	0.45	0.30	0.92
Skewness	−0.16	−0.29	0.76
Kurtosis	3.22	2.90	1.64
JB	0.38	0.00	0.49
***JB p*-value**	0.16	0.08	0.18
***JB*-stat**	12.16	3.56	67.32
Panel B	**Mean**	**Median**	**Std Dev**
Return	4.77	6.31	4.84
Std Dev	14.47	12.78	3.56
Sharpe	0.22	0.32	0.27
Skewness	−1.00	−1.03	0.27
Kurtosis	3.37	3.73	0.83
JB	1.00	1.00	0.00
***JB p*-value**	0.01	0.01	0.01
***JB*-stat**	15.32	14.44	6.61
Panel C	**Flows**		
Mean	−0.007		
Std Dev	0.14		
Skewness	−0.64		
Kurtosis	2.09		
JB	1.00		
***JB p*-value**	0.02		
***JB*-stat**	7.47		

Table 3.2: Properties of convertible bonds. The table reports convertible bonds' characteristics for each year from October 2002 to September 2009 (2002 includes 3 months and 2009 includes 9 months): average number of convertible bonds in the sample (*NumofCB*); cross-sectional mean of annualized monthly returns in percentage points (*Return*); 12-month average of annualized cross-sectional standard deviation in percentage points (*CSStd*); cross-sectional mean of annualized 12-month standard deviation in percentage points (*12mthStd*); average delta of convertible bonds in percentage points (*Delta*); average implied volatility in percentage points (*ImplVol*); average market capitalization of convertible bonds in USD mn (*MktCap*); and 12-month average of total market capitalization of convertible bonds in USD bn (*TotalMktCap*).

	NumofCB	***Return***	***CSStd***	***12mthStd***	***Delta***	***ImplVol***	***MktCap***	***TotalMktCap***
2002	292	32	35	21	40	46	583	179
2003	284	22	19	14	49	38	612	189
2004	254	13	22	17	55	31	636	173
2005	239	4	16	14	56	27	630	163
2006	250	10	17	13	61	28	671	178
2007	254	5	19	15	62	28	789	214
2008	238	−37	30	33	49	42	798	197
2009	189	47	29	22	43	51	692	136

Table 3.3: Conventional multifactor convertible bond model for the equally weighted and value-weighted portfolios. The factors are the following: *EMKT* is the excess equity market return (value-weighted return on all NYSE, AMEX, and NASDAQ stocks minus 1-month T-bill rate), *TERM* is the term spread between 10-year and 1-year Treasury yields, *DEF* is the default spread between *Baa* and *Aaa* Moody's corporate bond yields, *VIX* is the monthly percent change in the VIX index, and *REC* is a dummy variable on the NBER recession period from January 2007 through June 2009. The t-statistics appear in parentheses. Two stars denote a 5% level of significance. One star denotes a 10% level of significance.

	Equally weighted		**Value-weighted**	
	I	**II**	**I**	**II**
Const	−0.67	−1.20*	−0.66	−1.21**
	(−1.17)	(−1.87)	(−1.26)	(−2.06)
EMKT	0.63**	0.60**	0.58**	0.55**
	(8.66)	(8.03)	(8.74)	(8.09)
TERM	−0.03	−0.02	0.00	0.00
	(−0.13)	(−0.12)	(−0.01)	(0.01)
DEF	0.72*	1.47**	0.82**	1.59**
	(1.68)	(2.43)	(2.07)	(2.86)
VIX	−0.02	−0.02	−0.02	−0.02
	(−0.80)	(−0.94)	(−0.85)	(−1.01)
REC	—	−1.56*	—	−1.60*
	—	(−1.73)	—	(−1.94)
Adj R^2	0.66	0.67	0.66	0.68

Table 3.4: Hedge fund demand surprise. *HFD* is defined as OLS residuals from an autoregressive process $AR(4)$ for convertible arbitrage hedge fund outflows. Panel A reports regression coefficients with t-statistics in parentheses and regression diagnostic results. Panel B reports the correlation matrix between hedge fund demand surprise and conventional convertible bond factors: *EMKT* is the excess equity market return, *TERM* is the term spread between 10-year and 1-year Treasury yields, *DEF* is the default spread between Moody's *Baa* and *Aaa* corporate bond yields, and *VIX* is the monthly change in the VIX index.

Panel A	**Coefficient**
Const	-0.58^{*}
	(-1.94)
Outflows(-1)	0.40^{**}
	(3.93)
Outflows(-2)	0.06
	(0.64)
Outflows(-3)	0.58^{**}
	(6.16)
Outflows(-4)	-0.43^{**}
	(-4.11)
Adj R^2	0.44
Log-Likelihood	−177.00
Durbin-Watson	1.87
AIC	4.61
F**-statistic**	16.28

Panel B

Pearson Correlation	***EMKT***	***TERM***	***DEF***	***VIX***	***HFD***
EMKT	1.00	0.00	−0.16	−0.66	0.24
TERM	0.00	1.00	0.35	−0.12	−0.08
DEF	−0.16	0.35	1.00	−0.11	−0.28
VIX	−0.66	−0.12	−0.11	1.00	−0.11
HFD	0.24	−0.08	−0.28	−0.11	1.00

Kendall Correlation	***EMKT***	***TERM***	***DEF***	***VIX***	***HFD***
EMKT	1.00	0.04	0.02	−0.47	−0.04
TERM	0.04	1.00	0.19	−0.06	−0.03
DEF	0.02	0.19	1.00	−0.06	0.07
VIX	−0.47	−0.06	−0.06	1.00	0.04
HFD	−0.04	−0.03	0.07	0.04	1.00

Spearman Correlation	***EMKT***	***TERM***	***DEF***	***VIX***	***HFD***
EMKT	1.00	0.07	0.01	−0.65	−0.05
TERM	0.07	1.00	0.30	−0.09	−0.04
DEF	0.01	0.30	1.00	−0.09	0.08
VIX	−0.65	−0.09	−0.09	1.00	0.06
HFD	−0.05	−0.04	0.08	0.06	1.00

Table 3.5: Impact of unexpected (surprise) flows, inflows, and outflows on the returns of equally weighted and value-weighted portfolios of convertible bonds. The table reports OLS coefficients from time-series regressions of convertible bond portfolio returns on conventional convertible bond factors: *EMKT* is the excess equity market return (value-weighted return on all NYSE, AMEX, and NASDAQ stocks minus the 1-month T-bill rate); *TERM* is the interest-rate spread between 10-year and 1-year Treasury yields; *DEF* is the default spread between Moody's *Baa* and *Aaa* corporate bond yields; and *VIX* is the monthly change in the VIX index. We augment this regression by adding the surprise flows, inflows, and outflows *SurF*. Surprise is defined as OLS residuals of the autoregressive process $AR(p)$, where the number of lags p is chosen according to the AIC. The t-statistics appear in parentheses.

	Equally weighted			**Value-weighted**		
	Flows	**Inflows**	**Outflows**	**Flows**	**Inflows**	**Outflows**
Const	−0.08	−0.25	0.03	−0.12	−0.24	−0.04
	(−0.15)	(−0.43)	(0.05)	(−0.22)	(−0.45)	(−0.07)
EMKT	0.65**	0.63**	0.65**	0.60**	0.58**	0.60**
	(8.90)	(8.62)	(9.20)	(8.89)	(8.69)	(9.09)
TERM	−0.13	−0.12	−0.12	−0.11	−0.10	−0.10
	(−0.66)	(−0.62)	(−0.63)	(−0.57)	(−0.54)	(−0.54)
DEF	0.55	0.68	0.44	0.67*	0.77*	0.59
	(1.28)	(1.58)	(1.03)	(1.69)	(1.94)	(1.50)
VIX	−0.01	−0.02	−0.02	−0.01	−0.02	−0.02
	(−0.73)	(−0.80)	(−0.83)	(−0.79)	(−0.85)	(−0.88)
SurF	−0.14	0.01	−0.26**	−0.11	0.00	−0.20*
	(−1.57)	(0.06)	(−2.41)	(−1.32)	(−0.02)	(−1.93)
Adj $\boldsymbol{R^2}$	0.66	0.65	0.68	0.67	0.66	0.67

Table 3.6: Granger causality test. This table reports test statistics and critical values for the Granger causality test between surprise in flows, inflows, and outflows and the returns on equally weighted and value-weighted portfolios of convertible bonds. Granger causality is tested with 1 lag and a 5% significance level. F and G are the values of the F-statistic of the Granger test. CV is the critical value from the F-distribution. Rule: if $F > CV$, then we reject the null hypothesis that convertible bond returns do not cause flows; if $G > CV$, then we reject the null hypothesis that flows do not cause convertible bond returns.

	Equally weighted			**Value-weighted**		
	Flows	**Inflows**	**Outflows**	**Flows**	**Inflows**	**Outflows**
CV	3.96	3.96	3.96	3.96	3.96	3.96
F	0.62	1.43	1.52	0.39	1.35	1.20
G	2.90	3.66	3.60	4.47	4.31	4.86

Table 3.7: Descriptive statistics of *HFD* beta-sorted quantile portfolios of convertible bonds. The portfolio returns are the post-ranking out-of-sample next-month returns of the equally weighted portfolio with monthly rebalancing. The *HFD* beta is estimated using a fixed sample window of 24 months. *HFD* is the surprise in convertible arbitrage hedge fund outflows. The bottom quantile *Q1* contains convertible bonds with the lowest sensitivity while the top quantile *Q3* contains convertible bonds with the highest sensitivity. The price-pressure factor return (PP) is defined as the return spread between the top and bottom quantiles. The t-statistics and the Newey-West t-statistics (NW) are reported in parentheses. Panel A presents the distributional characteristics of the three quantile portfolios and the price-pressure factor return. Panel B presents the average characteristics of a convertible bond included in each of the three quantile portfolios: *HFD* beta and its Newey-West t-statistic, equity market delta, volatility, interest-rate spread, and market capitalization.

Panel A	***Q1***	***Q2***	***Q3***	***PP***
Mean	0.22	0.47	0.55	0.32
NW t-stat	(0.30)	(0.68)	(0.99)	(1.07)
t-stat	(0.41)	(0.84)	(1.17)	(1.39)
Median	0.50	0.93	0.70	0.43
Max	11.97	9.73	7.30	3.91
Min	−20.26	−18.52	−16.76	−6.74
Std Dev	4.20	4.27	3.59	1.79
Skewness	−1.84	−1.72	−1.91	−0.79
Kurtosis	12.15	9.34	10.85	5.72

Panel B	***Q1***	***Q2***	***Q3***
β_{HFD}	-1.14^{**}	0.00	1.21^{**}
NW t-stat	(−3.09)	(−0.02)	(2.64)
Delta	57.47	55.38	54.74
Hist Volatility	32.63	32.07	32.16
Spread	275.17	246.22	218.98
Market Cap	770.57	807.67	763.93

Table 3.8: OLS estimates of risk exposures for the *HFD* beta-sorted quantile portfolios (*Q1* stands for the bottom quantile, *Q2* for the middle quantile, and *Q3* for the top quantile) and the price-pressure factor (*PP*), defined as the return spread between the top and bottom quantile portfolios. Newey-West t-statistics are reported in parentheses. Two stars denote a 5% level of significance. One star denotes a 10% level of significance. Panel A reports the results for the four-factor convertible bond model. Panel B reports the results for the one-factor CAPM.

Panel A	***Q1***	***Q2***	***Q3***	***PP***
Const	-1.23^{**}	-0.65	-0.21	1.02^{**}
	(-2.72)	(-1.15)	(-0.35)	(2.35)
EMKT	0.66^{**}	0.58^{**}	0.56^{**}	-0.11^{*}
	(3.97)	(3.27)	(3.28)	(-1.72)
TERM	-0.06	-0.12	-0.30	-0.24
	(-0.19)	(-0.32)	(-0.96)	(-1.33)
DEF	1.00^{*}	0.78	0.70	-0.30
	(1.72)	(1.10)	(0.93)	(-0.67)
VIX	-0.02	-0.04	-0.01	0.00
	(-0.75)	(-1.61)	(-0.52)	(0.44)
Adj $\boldsymbol{R}^2$	0.63	0.60	0.62	0.08

Panel B	***Q1***	***Q2***	***Q3***	***PP***
Const	0.04	0.29	0.75^{**}	0.72^{**}
	(0.08)	(0.87)	(3.12)	(2.30)
EMKT	0.71^{**}	0.60^{**}	0.52^{**}	-0.19^{**}
	(5.47)	(4.12)	(5.84)	(-2.71)
Adj $\boldsymbol{R}^2$	0.57	0.53	0.59	0.11

Table 3.9: Fama-Macbeth estimates of price-pressure factor premia. The test portfolios are double-sorted by *HFD* beta and by convertible bond market capitalization. Values of the t-statistics and Newey-West t-statistics (NW) appear in parentheses. Two stars denote a 5% level of statistical significance. One star denotes a 10% level of statistical significance. Model I reports the premium estimates in the context of the one-factor CAPM, and Model II augments the CAPM by adding the price-pressure factor. Model III reports the premium estimates in the context of a four-factor convertible bond model (CB model). Model IV augments the four-factor convertible bond model by adding the price-pressure factor.

	CAPM		**CB model**	
	I	**II**	**III**	**IV**
λ_0	2.87**	2.89**	2.99**	2.26*
***t*-stat**	(4.12)	(3.78)	(3.44)	(1.92)
NW *t*-stat	(2.92)	(2.64)	(2.66)	(1.92)
λ_{EMKT}	−3.85**	−3.88**	−4.38**	−3.19
***t*-stat**	(−2.93)	(−2.72)	(−3.03)	(−1.60)
NW *t*-stat	(−2.03)	(−2.03)	(−2.01)	(−1.48)
λ_{TERM}	—	—	0.38	0.41
***t*-stat**	—	—	(0.81)	(0.86)
NW *t*-stat	—	—	(0.76)	(0.81)
λ_{DEF}	—	—	0.28	0.38*
***t*-stat**	—	—	(1.32)	(1.83)
NW *t*-stat	—	—	(1.12)	(1.61)
λ_{VIX}	—	—	3.91	2.95
***t*-stat**	—	—	(0.95)	(0.67)
NW *t*-stat	—	—	(1.10)	(0.82)
λ_{PP}	—	0.38	—	0.38
***t*-stat**	—	(1.35)	—	(1.55)
NW *t*-stat	—	(1.16)	—	(1.26)
Adj $\boldsymbol{R}^2$	0.07	0.14	0.07	0.16

Table 3.10: Impact of price pressure on convertible bond mutual funds. This table presents OLS regressions of the equally weighted portfolio returns of convertible bond mutual funds on the price-pressure risk factor PP and $PP{\cdot}PL$ (where PL is a dummy variable for the post-Lehman period from September 2008 to March 2009), controlling for conventional convertible bond factors. Panel A shows the results for the CAPM and the four-factor convertible bond model. Panel B presents similar results for the Fama-French and Carhart models. The t-statistics appear in parentheses.

Panel A	**CAPM**			**CB model**		
	I	**II**	**III**	**I**	**II**	**III**
Const	0.26	0.41	0.56	−0.74	0.02	−0.77
	(0.71)	(0.90)	(1.30)	(−0.84)	(0.02)	(−0.80)
EMKT	0.28**	0.19**	0.12	0.23**	0.13	0.10
	(3.49)	(2.01)	(1.25)	(2.01)	(0.99)	(0.86)
TERM	—	—	—	0.10	−0.44	−0.32
	—	—	—	(0.32)	(−0.85)	(−0.66)
DEF	—	—	—	0.67	0.74	1.36
	—	—	—	(1.01)	(0.84)	(1.62)
VIX	—	—	—	−0.03	−0.03	−0.02
	—	—	—	(−0.96)	(−0.87)	(−0.47)
PP	—	−0.94**	−0.69**	—	−0.91**	−0.56**
	—	(−3.57)	(−2.67)	—	(−3.31)	(−2.04)
PP · PL	—	—	−1.98**	—	—	−2.28**
	—	—	(−2.97)	—	—	(−3.23)
Adj R^2	0.12	0.26	0.35	0.12	0.25	0.36

Panel B	**Fama-French**			**Carhart**		
	I	**II**	**III**	**I**	**II**	**III**
Const	0.40	0.51	0.61	0.40	0.52	0.60
	(1.10)	(1.15)	(1.44)	(1.09)	(1.15)	(1.39)
EMKT	0.39**	0.32**	0.22**	0.38**	0.32**	0.19
	(4.46)	(2.85)	(1.92)	(3.84)	(2.76)	(1.59)
SMB	−0.32*	−0.17	−0.09	−0.32*	−0.16	−0.10
	(−1.89)	(−0.74)	(−0.44)	(−1.86)	(−0.72)	(−0.47)
HML	−0.29**	−0.39**	−0.30*	−0.29**	−0.38**	−0.33*
	(−2.05)	(−2.25)	(−1.81)	(−2.04)	(−2.08)	(−1.88)
MOM	—	—	—	−0.01	0.01	−0.05
	—	—	—	(−0.17)	(0.15)	(−0.56)
PP	—	−0.89**	−0.70**	—	−0.91**	−0.64**
	—	(−3.40)	(−2.69)	—	(−3.24)	(−2.24)
PP · PL	—	—	−1.69**	—	—	−1.81**
	—	—	(−2.50)	—	—	(−2.54)
Adj R^2	0.18	0.31	0.37	0.17	0.29	0.36

Table 3.11: Robustness check: price-pressure factor based on value-weighted portfolio. Descriptive statistics of *HFD* beta-sorted quantile portfolios of convertible bonds. The portfolio returns are the post-ranking out-of-sample next-month returns of the value-weighted portfolios with monthly rebalancing. The *HFD* beta is estimated using a fixed sample window of 24 months. *HFD* is the surprise in convertible arbitrage hedge fund outflows. The bottom quantile *Q1* contains convertible bonds with the lowest sensitivity while the top quantile *Q3* contains convertible bonds with the highest sensitivity. The price-pressure factor return (*PP*) is defined as the return spread between the top and bottom quantiles. Values of the t-statistics and Newey-West t-statistics appear in parentheses. Panel A presents the distributional characteristics of the three quantile portfolios and the price-pressure factor return; panel B presents the average characteristics of a convertible bond included in each of the three quantile portfolios: *HFD* beta and its Newey-West t-statistic, equity market delta, volatility, interest-rate spread, and market capitalization.

Panel A	***Q1***	***Q2***	***Q3***	***PP***
Mean	0.29	0.54	1.01**	0.71**
NW *t*-stat	(0.37)	(0.91)	(1.84)	(1.62)
***t*-stat**	(0.49)	(1.04)	(2.35)	(2.07)
Median	0.35	1.08	1.13	0.78
Max	−19.59	−16.04	−11.15	−8.15
Min	−20.26	−18.52	−16.76	−6.74
Std Dev	4.60	4.00	3.29	2.65
Skewness	−0.95	−1.35	−0.65	−0.60
Kurtosis	9.47	6.96	5.38	6.21

Panel B	***Q1***	***Q2***	***Q3***
$\beta_{\boldsymbol{HFD}}$	−0.89**	−0.01	0.88**
NW *t*-stat	(−2.79)	(−0.07)	(2.64)
Delta	57.47	55.38	54.74
Hist Volatility	32.63	32.07	32.16
Spread	275.17	246.22	218.98
Market Cap	770.57	807.67	763.93

Table 3.12: Robustness check: price-pressure factor based on value-weighted portfolio. The table reports the OLS estimates of risk exposures for the *HFD* beta-sorted quantile portfolios (*Q1* stands for the bottom quantile, *Q2* for the middle quantile, and *Q3* for the top quantile) and the price-pressure factor (*PP*), defined as the return spread between the top and bottom quantile portfolios. The portfolio returns are the post-ranking out-of-sample next-month returns of the value-weighted portfolios with monthly rebalancing. Newey-West t-statistics are reported in parentheses. Two stars denote a 5% level of significance. One star denotes a 10% level of significance. Panel A reports the results for the four-factor convertible bond model. Panel B reports the results for the one-factor CAPM.

	Q1	***Q2***	***Q3***	***PP***
Const	-1.60^{**}	-0.64	-0.28	1.32^{**}
	(-3.22)	(-1.06)	(-0.53)	(2.78)
EMKT	0.68^{**}	0.51^{**}	0.51^{**}	-0.18^{**}
	(4.18)	(2.75)	(3.63)	(-2.09)
TERM	-0.08	-0.21	-0.27	-0.19
	(-0.21)	(-0.63)	(-0.99)	(-0.76)
DEF	1.36^{**}	0.92	1.08^{*}	-0.28
	(2.33)	(1.32)	(1.75)	(-0.60)
VIX	-0.03	-0.05	-0.01	0.01
	(-1.06)	(-1.45)	(-0.53)	(0.86)
Adj R^2	0.61	0.56	0.62	0.10

Panel B	***Q1***	***Q2***	***Q3***	***PP***
Const	0.04	0.29	0.75^{**}	0.72^{**}
	(0.08)	(0.87)	(3.12)	(2.30)
EMKT	0.71^{**}	0.60^{**}	0.52^{**}	-0.19^{**}
	(5.47)	(4.12)	(5.84)	(-2.71)
Adj R^2	0.57	0.53	0.59	0.11

Table 3.13: Robustness check: price-pressure factor based on value-weighted portfolio. The table reports the impact of the price-pressure factor on the returns of convertible bond mutual funds. The returns of equally weighted portfolio of mutual funds are regressed on the price-pressure risk factor PP and $PP \cdot PL$ (where PL is a dummy variable for the post-Lehman period from September 2008 to March 2009), using OLS and controlling for conventional convertible bond factors. Panel A shows the results for the CAPM and the four-factor convertible bond model. Panel B presents similar results for the Fama-French and Carhart models. The t-statistics appear in parentheses.

Panel A	**CAPM**			**CB model**		
	I	**II**	**III**	**I**	**II**	**III**
Const	0.26	0.53	0.64	−0.74	−0.17	−0.69
	(0.71)	(1.12)	(1.37)	(−0.84)	(−0.17)	(−0.68)
EMKT	0.28**	0.17*	0.08	0.23**	0.13	0.06
	(3.49)	(1.71)	(0.72)	(2.01)	(0.94)	(0.45)
TERM	—	—	—	0.10	−0.33	−0.17
	—	—	—	(0.32)	(−0.63)	(−0.33)
DEF	—	—	—	0.67	0.86	1.20
	—	—	—	(1.01)	(0.96)	(1.37)
VIX	—	—	—	−0.03	−0.03	−0.02
	—	—	—	(−0.96)	(−0.76)	(−0.47)
PP	—	−0.60**	−0.45**	—	−0.56**	−0.38*
	—	(−3.18)	(−2.36)	—	(−2.92)	(−1.89)
$PP \cdot PL$	—	—	−0.91**	—	—	−1.07**
	—	—	(−2.14)	—	—	(−2.37)
Adj R^2	0.12	0.23	0.28	0.12	0.22	0.28

Panel B	**Fama-French**			**Carhart**		
	I	**II**	**III**	**I**	**II**	**III**
Const	0.40	0.60	0.67	0.40	0.58	0.65
	(1.10)	(1.27)	(1.45)	(1.09)	(1.23)	(1.39)
EMKT	0.39**	0.30**	0.20	0.38**	0.29**	0.17
	(4.46)	(2.54)	(1.54)	(3.84)	(2.32)	(1.19)
SMB	−0.32*	−0.20	−0.17	−0.32*	−0.21	−0.18
	(−1.89)	(−0.88)	(−0.74)	(−1.86)	(−0.90)	(−0.78)
HML	−0.29**	−0.34*	−0.28	−0.29**	−0.37*	−0.32*
	(−2.05)	(−1.93)	(−1.56)	(−2.04)	(−1.94)	(−1.71)
MOM	—	—	—	−0.01	−0.03	−0.07
	—	—	—	(−0.17)	(−0.38)	(−0.77)
PP	—	−0.53**	−0.43**	—	−0.51**	−0.38*
	—	(−2.80)	(−2.19)	—	(−2.64)	(−1.90)
$PP \cdot PL$	—	—	−0.74*	—	—	−0.82*
	—	—	(−1.71)	—	—	(−1.83)
Adj R^2	0.18	0.26	0.29	0.17	0.25	0.28

Table 3.14: Robustness check: Fama-Macbeth estimates of price-pressure factor premia. The price-pressure factor return is defined as the return spread between the top and bottom equally weighted quantile portfolios. The test portfolios are double-sorted by the *HFD* beta and by the convertible bond delta. Values of the t-statistics and Newey-West t-statistics appear in parentheses. Two stars denote a 5% level of significance. One star denotes a 10% level of significance. Model I reports the premium estimates in the context of the one-factor CAPM, and Model II augments the CAPM by adding the price-pressure factor. Model III reports the premium estimates in the context of a four-factor convertible bond model (CB model). Model IV augments the four-factor convertible bond model by adding the price-pressure factor.

	CAPM		**CB model**	
	I	**II**	**III**	**IV**
λ_0	0.46	0.14	0.53*	−0.07
***t*-stat**	(1.42)	(0.43)	(1.65)	(−0.21)
NW *t*-stat	(1.40)	(0.39)	(1.69)	(−0.20)
λ_{EMKT}	−0.06	0.67	−0.01	0.87
***t*-stat**	(−0.07)	(0.77)	(−0.01)	(0.92)
NW *t*-stat	(−0.06)	(0.73)	(0.00)	(0.95)
λ_{TERM}	—	—	0.26	0.47
***t*-stat**	—	—	(0.67)	(1.13)
NW *t*-stat	—	—	(0.60)	(0.97)
λ_{DEF}	—	—	−0.21	0.17
***t*-stat**	—	—	(−0.97)	(0.81)
NW *t*-stat	—	—	(−0.77)	(0.64)
λ_{VIX}	—	—	−3.47	−3.78
***t*-stat**	—	—	(−0.66)	(−0.72)
NW *t*-stat	—	—	(−0.76)	(−0.83)
λ_{PP}	—	0.46*	—	0.34
***t*-stat**	—	(1.73)	—	(1.26)
NW *t*-stat	—	(1.28)	—	(1.03)
$Adj\ R^2$	0.19	0.29	0.27	0.29

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